Contrapositives

If statement A, then statement B.

contrapositive:
If NOT Statement B, then NOT statement A.

A

Example: If you drive the 60 freeway westbound, then you will be in L.A. B

contrapositive: If you will not be in Los Angeles, then you do not drive the 60 freeway westbound.

CONVERSE:
If statement B, then statement A.

converse: If you will be in Los Angeles, then you drive the 60 freeway westbound.

INVERSE:
If NOT statement A, then NOT statement B.

Inverse: If you do not drive the 60 freeway westbound, then you will not be in LA.

Let a, b, n be integers. If ab is NOT an integer multiple of n, then a is NOT a multiple of n and b is NOT a multiple of n.

the negation of "and" is "or" b/c of De Morgan's law (Math 191)

CONTRAPOSITIVE:

If a is a multiple of n or b is a multiple of n, then ab is a multiple of n.

Proof (of contrapositive):

- Suppose a is an integer multiple of n. Then there exists an integer k that satisfies

$$a = kn.$$

So we have

$$\begin{aligned} ab &= (kn)b \\ &= n(kb) = (kb)n \end{aligned}$$

Since kb is also an integer, we conclude that ab is a multiple of n.

- Suppose b is an integer multiple of n. Then there exists an integer ℓ that satisfies

$$b = \ell n$$

So we have

$$\begin{aligned} ab &= a(\ell n) \\ &= n(a\ell) = (a\ell)n \end{aligned}$$

Since $a\ell$ is also an integer, we conclude that ab is an integer multiple of n

$\blacksquare$