7/17/2019 Discussion 4

Contrapositives

If statement A, then statement B.

- CONTRAPOSITIVE:
  If NOT statement B, then NOT statement A.

Eg If you drive the 60 freeway west-bound, then you will be in Los Angeles

→ contrapositive: If you will not be in Los Angeles, then you do not drive the 60 freeway westbound.

- CONVERSE:
  If statement B, then statement A.

→ converse: If you will be in Los Angeles, then you drive the 60 freeway westbound.

- INVERSE:
  If NOT statement A, then NOT statement B.

→ inverse: If you do not drive the 60 freeway westbound, then you will not be in Los Angeles.

Let a, b, n be integers. If ab is NOT an integer multiple of n, then a is NOT a multiple of n and b is NOT a multiple of n.

CONTRAPOSITIVE:
If a is a multiple of n or b is a multiple of n, then then ab is a multiple of n.

the negation of "and" is "or" because of De Morgan's Law (Math 144)

Proof (of contrapositive):

- Suppose a is a multiple of n. Then there exists an integer k that satisfies

$$a = kn.$$

So we have

$$ab = (kn)b = n(kb) \quad \text{or} \quad (kb)n.$$

since kb is also an integer, we conclude that ab is a multiple of n.

- Suppose b is an integer multiple of n. Then there exists an integer l that satisfies.

$$b = ln$$

so we have $ab = a(ln)$

$(al)n$

since al is also an integer, we conclude that ab is an integer multiple of n.