07-17-19 | DISCUSSION

Contrapositives

If statement A, then statement B.

CONTRAPOSITIVE:

If NOT statement B, the NOT statement A

Example: If you drive the 60 freeway westbound
the you will be in Los Angeles

contrapositive: If you will not be in Los Angeles,
the you do not drive the 60 freeway westbound

CONVERSE:

If statement B, then statement A

converse: If you will be in Los Angeles.
then you drive the 60 freeway westbound

INVERSE:

If NOT statement A, then NOT statement B

inverse: If you do not drive the 60 freeway westbound
the you will not be in Los Angeles

Let a, b, n be integers. If ab is NOT an integer multiple of n, then a is NOT a multiple of n and b is NOT a multiple of n.

CONTRAPOSITIVE:

If a is a multiple of n (or) b is a multiple of n, then ab is a multiple of n.

the negation of "and" is "or" because of De Morgan's Law (MATH 144)

Proof: (of contrapositive):

- Suppose a is a multiple of n. Then there exists an integer k that satisfies

$$a = kn$$

So we have

$$ab = (kn)b$$
$$= n(kb)$$

since kb is also an integer, we conclude that ab is a multiple of n

- Suppose b is an integer multiple of n. Then there exists an integer ℓ that satisfies

$$b = \ell n$$

So we have

$$ab = a(\ell n)$$

$$= n(a\ell)$$

since $a\ell$ is also an integer, we conclude that ab is an integer multiple of n.