

Discussion #4

Contrapositives

If Statement A, then Statement B.

Contrapositive:

If NOT Statement B, then NOT Statement A.

Example: If ^{Statement A} you drive ^{there} on 60 freeway westbound, then ~~not~~ ^{Statement B} you will be in Los Angeles.

Contrapositive:

If you will not be in Los Angeles, then you do not drive the 60 freeway westbound.

Converse:

If you will be in Los Angeles, then you drive the 60 freeway westbound.

Converse:

If Statement B, then Statement A.

Inverse:

If you do not drive the 60 freeway westbound, then you will not be in Los Angeles.

Inverse:

If NOT Statement A, then NOT Statement B.

Ex: Let a, b, n be integers. If ab is ~~not a multiple of n~~ , NOT an integer multiple of n , then a is not a multiple of n and b is NOT a multiple of n .

Contrapositive:

If a is a multiple of n ^{or} b is a multiple of n , then ab is a multiple of n .
↓ The negation of "and" is "or", because of De Morgan's Law (MATH 144)

Proof: (of Contrapositive):

- Suppose a is ^{integer} ~~only~~ multiple of n . Then there exists an integer k ~~such~~ that satisfies $a = kn$.

So we have $ab = (kn)b$
 $= \cancel{kn} \cdot n(kb)$

Since $\cancel{kb}$ is also an integer, we conclude that ab is an ^{integer} multiple of n .

- Suppose b is an integer multiple of n , Then there exist Integer l that satisfies $\cancel{b=ln} \quad b=ln$

So we have $ab = a(ln)$
 $= n(al)$

Since al is also an integer, we conclude that ab is an integer ~~that~~ multiple of n .