Dis. 7/24

Induction

Ex 1: prove $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

Proof: For $n=1$ we have $1 = \frac{1\times 2}{2} = 1$ $\rightarrow$ true statement.

for $n=k$ we assume $1+2+\cdots+k = \frac{k(k+1)}{2}$

for $n=k+1$ $1+2+\cdots+k+(k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{(k+2)(k+1)}{2} = \frac{n(n+1)}{2}$

qed.

EX2: Prove $1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$P(0)$: $1^2 = \frac{1(1+1)(2\times1+1)}{6} = \frac{1\times2\times3}{6} = 1$

$P(0)$ is true

So now assume $P(k)$ is true

which is $1^2+2^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$

Now for $n=k+1$ $1^2+\cdots+k^2+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$$= \frac{k(k+1)(2k+1)+6((k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1)+6(k+1))}{6}$$

$$= \frac{(k+1)(2k^2+k+6k+6)}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

qed.

EX3: Prove $1+3+\cdots+(2n-1) = n^2$

Proof: $P(0)$ For $n=1$ $2(1)-1 = 1^2$ is true.

Assume $P(k)$ is true : $1+\cdots+(2k-1) = k^2$

So for $n=k+1$ $1+3+\cdots+(2k-1)+(2(k+1)-1) = k^2+(2k+1) = (k+1)^2 = n^2$

qed.