Discussion notes

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Induction:

Ex 1) Prove:

$$1+2+3+\dots+n = \frac{n(n+1)}{2} \quad \forall \text{ positive integer } n$$

Proof: We will prove by induction.

Base step: for $n=1$, $1 = \frac{(1)(1)+1}{2}$

$1=1$

Induction step: for $n=k$,

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

prove $n=k+1$

$$1+2+3+\dots+k(+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

Ex 2) Prove

$$1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$\forall$ positive int. n

Base step: Proof: For $n=1$ $1^2 = \frac{(1)((1)+1)(2(1)+1)}{6} = 1$

Inductive step: For $n=k$, $1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$

For $n=k+1$, we have

$$1^2+2^2+3^2+\dots+(k+1)^2 = (1^2+2^2+3^2+\dots+k^2)+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6} = \frac{(k+1)(2k^2+k+6k+6)}{6}$$

$$= \frac{(k+1)(2k+3)(k+2)}{6} = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Ex 3) Prove

$$1+3+5+7+\dots+(2n-1) = n^2$$

$\forall$ positive ints. n.

Proof: Base step:

$2(1)-1 = (1)^2$

$1=1$

Inductive step for $n=k$,

$$1+3+5+7+\dots+(2k-1)=k^2$$

for $n=k+1$, we have

$$1+3+5+7+\dots+(2(k+1)-1)=1+3+5+7+\dots+(2k+1)$$

$$=(1+3+5+7+\dots+(2k-1))+(2k+1)$$

$$=k^2+(2k+1)$$

$$=k^2+2k+1$$

$$=(k+1)^2.$$