DISCUSSION

Induction

Ex 1: Prove that we have

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$ for all positive integers n

Proof: We will prove by induction

Base Step: For $n=1$, we have

$$1 = \frac{1(1+1)}{2}$$

$$1 = 1$$

Induction Step: For $n=k$, we assume
(we assume for $n=k$)

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

We will prove the statement for $n=k+1$. We have

$$1+2+3+\dots+(k+1) = (1+2+3+\dots+k)+(k+1)$$

$$= \frac{k(k+1)}{2}+(k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\blacksquare$

Ex 2: Prove that we have

$$1^2+2^2+3^2+\dots+n^2=\frac{n(n+1)(2n+1)}{6}$$

for all positive integers n.

Proof: For $n=1$, we have

Base Step:

$$1^2=\frac{(1)((1)+1)(2(1)+1)}{6}$$

$$1=1$$

Induction step:

For $n=k$, we assume

$$1^2+2^2+3^2+\dots+k^2=\frac{k(k+1)(2k+1)}{6}$$

For $n=k+1$, we have

$$1^2+2^2+3^2+\dots+(k+1)^2=\left(1^2+2^2+3^2+\dots+k^2\right)+(k+1)^2$$

$$=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

$$=\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$=(k+1)\left(\frac{k(2k+1)+6(k+1)}{6}\right)$$

$$=\frac{(k+1)(2k^2+k+6k+6)}{6}$$ $(2k+3)(k+2)$

$$=(k+1)(2k^2+7k+6$$

$$=\frac{(k+1)(2k+3)(k+2)}{6}$$

$$=\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

$\square$

Ex 3: Prove that we have

$$\overset{2(1)-1}{1}+\overset{2(2)-1}{3}+\overset{2(3)-1}{5}+\overset{2(4)-1}{7}+\ldots+(2n-1)=n^2$$

for all positive integers n.

Proof: Base step: For $n=1$, we have

$$2(1)-1=(1)^2$$
$$1=1$$

Induction step: For $n=k$, we assume

$$1+3+5+7+\ldots+(2k-1)=k^2$$

For $n=k+1$, we have

$$1+3+5+7+\ldots+(2(k+1)-1)=1+3+5+7+\ldots+(2k+1)$$

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$= (1+3+5+7+\ldots+(2k-1)+(2k+1)$

$= k^2+(2k+1)$

$= k^2+2k+1$

$= (k+1)^2$

$\boxed{}$

<u>Induction</u> (in a nutshell)

1. Prove the statement for $n=1$
2. Assume the statement for $n=k$
3. Prove the statement for $n=k+1$