7/24/2019 **Disscussion Week 5**

## Induction

Example ① : Prove that we have

$$1+2+3+\cdots+n = \frac{n(n+1)}{2} \quad \text{for all positive integers } n.$$

proof: We will prove by induction.

① Base Step: For $n=1$, we have

$$1 = \frac{(1)((1)+1)}{2}$$

$$1 = 1$$

② Induction step: For $n=k$, we assume (we assume for $n=k$)

$$1+2+\cdots+k = \frac{k(k+1)}{2}.$$

We will prove the statement for $n=k+1$,

we have

$$1+2+3+\cdots+(k+1) = (1+2+3+\cdots+k)+(k+1)$$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\boxed{\phantom{x}}$

Example ②: Prove that we have

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all positive integers $n$.

proof: For $n=1$, we have

① Base Step

$$1^2 = \frac{(1)((1)+1)(2(1)+1)}{6}$$

$$1 = 1$$

① Induction Step For $n=k$, we assume

$$1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

For $n=k+1$, we have

$$1^2+2^2+3^2+\cdots+(k+1)^2 = (1^2+2^2+3^2+\cdots+k^2)+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1)+6(k+1))}{6}$$

$$= \frac{(k+1)((2k^2+k)+(6k+6))}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$[(2k+3)(k+2) = 2k^2+4k+3k+6 = 2k^2+7k+6]$$

$$= \frac{(k+1)(2k+3)(k+2)}{6}$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

$\boxed{\phantom{x}}$

Example ③ Prove that we have

$$\overset{2(1)-1}{1} + \overset{2(2)-1}{3} + \overset{2(3)-1}{5} + \overset{2(4)-1}{7} + \cdots + (2n-1) = n^2$$

for all positive integers $n$.

① Base Step: For $n=1$, we have

$$2(1)-1 = (1)^2$$

$$1 = 1$$

② Induction Step For $n=k$, we assume

$$1+3+5+7+\cdots+(2k-1) = k^2$$

For $n=k+1$,

we have

→

$$1+3+5+7+\cdots+(2(k+1)-1) = 1+3+5+7+\cdots+(2k+1)$$
$$= (1+3+5+7+\cdots+(2k-1)) + (2k+1)$$
$$= k^2 + (2k+1)$$
$$= k^2+2k+1$$
$$= (k+1)^2$$

Induction (in a nut shell)

1. PROVE the statement for $n=1$.
2. ASSUME the statement for $n=k$.
3. PROVE the statement for $n=k+1$.