

Discussion 7/24/19 week 5

Induction

Example 1: Prove that we have $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Proof: for all positive integers n .

We will prove by induction.

Base step: for $n=1$, we have

$$\frac{(1+1)(1+1+1)}{2} = \frac{1(1+1)}{2}$$

$$1 = 1$$

Induction step: for $n=k$, we assume

(we assume for $n=1$)

$$1+2+3+\dots+k = k(k+1)$$

$$\text{we have } \frac{(k+1)(k+2)}{2}$$

We will prove the statement for $n=k+1$. We have

$$1+2+3+\dots+(k+1) = (1+2+3+\dots+k) + (k+1)$$

$$= \underline{k(k+1)} + (k+1)$$

$$\sim \text{using base } 2 \text{ result from previous step}$$

$$(k(k+1)) + 2(k+1) + 2 + k^2 + k + 2k + 2$$

$$= 2(k^2 + 3k + 2)$$

$$= (k+1)(k+2)$$

number 9, 4-17-102 result

$3 \cdot 4 = (1+2) \cdot 3 \cdot 4 \end{proof}$

Example 2: Prove that we have $1+2+3+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$(1+1^2) + (1+2^2) + (1+3^2) + \dots + (1+n^2) = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n .

Proof: Base step: for $n=1$, we have

$$1+1^2+1^2 = 1^2 = \underline{(1)(1+1)(2(1)+1)}$$

$$3(1+1) = 6$$

$$\sim \text{using base } 1 = 1 \text{ result from previous step}$$

Induction step: for $n=k$, we assume

$$1+2^2+3^2+\dots+k^2 = \underline{k(k+1)(2k+1)}$$

$$1+2^2+3^2+\dots+(k+1)^2 = \underline{(k+1)(k+2)(2k+3)}$$

for $n=k+1$, we have

$$\begin{aligned}
 1^2 + 2^2 + 3^2 + \dots + (k+1)^2 &= (1^2 + 2^2 + 3^2 + \dots + k^2) + (k+1)^2 \\
 (\text{Let } n = 1 + 2 + \dots + k + 1) &= k(k+1)(2k+1) + (k+1)^2 \\
 &= k(k+1)(2k+1) + 6(k+1)^2 \\
 &= (k+1)(k(2k+1) + 6(k+1)) \\
 &= (k+1)((2k^2+k) + (6k+6)) \\
 &= (k+1)(2k^2+7k+6) \\
 &= (k+1)(2k+3)(k+2)
 \end{aligned}$$

Example 3: prove that we have $1+3+5+7+\dots+(2n-1)=n^2$ for all positive integers n
base step: $2(1)-1=1^2$ for $n=1$, we have

Induction step: For $n=k$, we assume $1+3+5+7+\dots+(2k-1)=k^2$

$$\begin{aligned}
 \text{for } n=k+1 \text{ we have} \\
 1+3+5+7+\dots+(2(k+1)-1) &\approx 1+3+5+7+\dots+(2k-1)+(2k+1) \\
 &= (1+3+5+7+\dots+(2k-1))+(2k+1) \\
 &= k^2+(2k+1) \\
 &= k^2+2k+1 \\
 &= (k+1)^2
 \end{aligned}$$

Induction in a nutshell: 1. base proof ~

1. Prove the statement for $n=1$ ~~no induction~~
2. Assume the statement $n=k$ ~~is true~~
3. Prove the statement for $n=k+1$