Discussion 7/24/19 week 5

Induction

Example 1: Prove that we have $1+2+3+\dots+n = \frac{n(n+1)}{2}$ for all positive integers n.

Proof: We will prove by induction

Base step: for n=1, we have

$$1 = \frac{(1)((1)+(1))}{2}$$

$$1 = 1$$

Induction step: for n=k, we assume (we assume for n=k)

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

We will prove the statement for n=k+1. We have

$$1+2+3+\dots+(k+1) = (1+2+3+\dots+k)+(k+1)$$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2} \qquad k^2+k+2k+2$$

$$k^2+3k+2$$

$$= \frac{(k+1)(k+2)}{2}$$

~ end proof ~

Example 2: Prove that we have $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n.

Proof: Base step: for n=1, we have

$$1^2 = \frac{(1)(1+1)(2(1)+1)}{6}$$

$$1 = 1$$

Induction step: for n=k, we assume

$$1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

for n=k+1, we have

$$1^2+2^2+3^2+\dots+(k+1)^2 = (1^2+2^2+3^2+\dots+k^2)+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1)+6(k+1))}{6}$$

$$= \frac{(k+1)((2k^2+k)+(6k+6))}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)((2k+3)(k+2))}{6}$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

~ end proof ~

Example 3: prove that we have $1+3+5+7+\dots+(2n-1) = n^2$ for all positive integers n

proof: <u>base step</u>: $2(1)-1 = 1^2$ for $n=1$, we have

$1 = 1$

<u>Induction step</u>: for $n=k$, we assume

$1+3+5+7+\dots+(2k-1) = k^2$

for $n=k+1$, we have

$1+3+5+7+\dots+(2(k+1)-1) = 1+3+5+7+\dots+(2k-$

$= (1+3+5+7+\dots+(2k-1)) + (2k$

$= k^2+(2k+1)$

$= k^2+2k+1$

$= (k+1)^2$

~ end proof ~

Induction in a nutshell

1. Prove the statement for $n=1$
2. Assume the statement $n=k$
3. Prove the statement for $n=k+1$