07-24-19 DISCUSSION

Induction

Example 1 : Prove that we have

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$ for all positive integers, n.

Proof: we will prove by induction.

Base step: For n=1, we have

$$1 = \frac{(1)((1)+1)}{2}$$

$$1=1$$

Induction step: For n=k, we assume

$$1+2+3+\cdots+k = \frac{k(k+1)}{2}$$

we will prove the statement for n=k+1, we have

$$1+2+3+\cdots+(k+1) = (1+2+3+\cdots+k)+(k+1)$$
$$= \frac{k(k+1)}{2}+(k+1)$$
$$= \frac{k(k+1)+2(k+1)}{2}$$
$$= \frac{(k+1)(k+2)}{2}$$ $\blacksquare$

Example 2 : Prove that we have

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all positive integers, n.

Proof: For n=1, we have

Base step

$$1^2 = \frac{(1)\overset{2}{((1)+1)}\overset{3}{(2(1)+1)}}{6}$$

$$1=1$$

Induction step

For n=k, we assume

$$1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

For n=k+1, we have

$$1^2+2^2+3^2+\cdots+(k+1)^2 = (1^2+2^2+3^2+\cdots+k^2)+(k+1)^2$$
$$= \frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1)+6(k+1))}{6}$$

$$= \frac{(k+1)((2k^2+k)+(6k+6))}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)((2k+3)(k+2))}{6}$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

$\blacksquare$

Example 3: Prove that we have

$$1+3+5+7+\cdots+(2n-1) = n^2$$

for all positive integers n.

Proof: <u>Base step</u>: $2(1)-1 = 1^2$ $\leftarrow$ For $n=1$, we have

$1 = 1$

<u>Induction step</u>: For $n=k$, we asume

$$1+3+5+7+\cdots+2k-1 = k^2$$

For $n=k+1$, we have

$$1+3+5+7+\cdots+(2(k+1)-1) = 1+3+5+7+\cdots+(2k+1)$$
$$= (1+3+5+7+\cdots+(2k-1)) + (2k+1)$$
$$= k^2 + (2k+1)$$
$$= k^2 + 2k + 1$$
$$= (k+1)^2$$

$\blacksquare$

Induction (in a nutshell)

1. Prove the statment for $n=1$
2. Assume the statement for $n=k$
3. Prove the statement for $n=k+1$.