~~Induct~~ Discussion #5

Induction

Example 1: Prove that we have $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ for all positive integers n.

Proof: We will prove by induction.

Base step: For $n=1$, we have

$$1 = \frac{1(1+1)}{2}$$
$$1 = 1$$

Induction step: For $n=k$, we assume $1+2+3+\cdots+k = \frac{k(k+1)}{2}$

We will prove the statement for $n=k+1$. We have

$$1+2+3+\cdots+(k+1) = (1+2+3+\cdots+k)+(k+1)$$
$$= \frac{k(k+1)}{2} + (k+1)$$
$$= \frac{k(k+1)+2(k+1)}{2}$$
$$= \frac{(k+1)(k+2)}{2}$$

Example 2: Prove that we have $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n.

Proof: Base step: For $n=1$, we have

$$1^2 = \frac{(1)(\overset{2}{(1)+1})(\overset{3}{2(1)+1})}{6}$$
$$1 = 1$$

Induction step:

For $n=k$, we assume $1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$

For $n=k+1$, we have

$$1^2+2^2+3^2+\cdots+(k+1)^2 = \cancel{(1^2+2^2+3^2+}\cdots+k^2)+(k+1)^2$$
$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$
$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$
$$= \frac{(k+1)(k(2k+1)+6(k+1))}{6}$$
$$= \frac{(k+1)((2k^2+k)+(6k+6))}{6} \leftarrow = \frac{(k+1)(2k^2+7k+6)}{6}$$
$$= \frac{(k+1)((2k+3)(k+2))}{6} = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

Example 3: Prove that we have $1+3+5+7+\cdots+(2n-1)=n^2$ for all positive integers n.

$\overset{2(1)-1}{1}+\overset{2(2)-1}{3}+\overset{2(3)-1}{5}+7+\cdots+(2n-1)=n^2$

Proof: Base step: ~~$2(1)-1=(1)^2$~~

For $n=1$, we have

$$2(1)-1=(1)^2$$
$$1=1$$

Induction step: For $n=k$, we assume $1+3+5+7+\cdots+(2k-1)=k^2$.

For $n=k+1$, we have

$$\begin{aligned}1+3+5+7+\cdots+(2(k+1)-1) &= 1+3+5+7+\cdots+(2k+1)\\ &= (1+3+5+7+\cdots+(2k-1))+(2k+1)\\ &= k^2+(2k+1)\\ &= k^2+2k+1\\ &= (k+1)^2.\end{aligned}$$

Induction (in nutshell):

1. PROVE the statement for $n=1$.
2. ASSUME the statement for $n=k$.
3. PROVE the statement for $n=k+1$.