Union: $A \cup B$

Intersection: $A \cap B$

Proof: • First we will prove $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$

Let $x \in (A \setminus B) \cup (C \setminus B)$

Then $x \in (A \setminus B)$ or $x \in (C \setminus B)$

If $x \in A \setminus B$, then $x \in A$ and $x \notin B$

Since $A \subset A \cup C$ we have $x \in A \cup C$ and $x \notin B$

If $x \in C \setminus B$ then $x \in C$ and $x \notin B$

Since $C \subset A \cup C$ we have $x \in A \cup C$ and $x \notin B$

In either case, we have $x \in A \cup C$ and $x \notin B$,

so $x \in (A \cup C) \setminus B$

Therefore, $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$.

• Next, we will prove $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

Let $x \in (A \cup C) \setminus B$. Then $x \in A \cup C$ and $x \notin B$.

so we have $x \in A$ and $x \notin B$, or $x \in C$ and $x \notin B$.

So $x \in A \setminus B$, or $x \in C \setminus B$

$\therefore x \in (A \setminus B) \cup (C \setminus B)$

$\therefore (A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

$\therefore (A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B \Leftarrow$ two sets contained each other.

Universal quantifier:

For all, for every, for each. $\forall$

Existential quantifier:

There exists   for some   for one $\exists$

Examples:

Definition of continuity.

- For all $\varepsilon > 0$, there exists $\delta_\varepsilon > 0$ such that, for all $x, c \in \mathbb{R}$ with c fixed, if $|x-c| < \delta$, then $|f(x)-f(c)| < \varepsilon$
- For all students in Math 131 this summer, there exists an instructor running the course.

Odd and even integers.

Let a and b be even integers.

Prove that $a+b$, $a-b$, ab are also even integers.