Discussion 6/26/19 week 1

union: $x \in A \cup B = x \in A$ or $x \in B$

intersection: If $x \in A \cap B$, then $x \in A$ & $x \in B$

If $x \in A$ implies $x \in B$, then $A \subset B$

A is contained in B

set containment proofs

Example 1: Prove $(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$

If $A \subset B$ & $B \subset A$, then $A =$ B

$A \setminus B = \{x \in A : x \notin B\}$

Proof:

first we will prove

$(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$

Let $x \in (A \setminus B) \cup (C \setminus B)$ Then $x \in A \setminus B$ or $x \in C \setminus B$

If $x \in A \setminus B$, then $x \in A$ & $x \notin B$. since $A \subset A \cup C$, we have $x \in A \cup C$ & $x \notin B$

If $x \in C \setminus B$, then $x \in C$ & $x \in B$. since $C \subset A \cup C$, we have $x \in A \cup C$ & $x \in B$

In either case we have $x \in A \cup C$ & $x \notin B$ so $x \in (A \cup C \setminus B$

Therefore, $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$

Next we will prove $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

Let $x \in (A \cup C) \setminus B$. Then $x \in A \cup C$ & $x \notin B$

so we have $x \in A$ or $x \in C$, & $x \in B$,

& so $x \in A$ & $x \in B$, or $x \in C$ & $x \in B$

so $x \in A \setminus B$, or $x \in C \setminus B$

Therefore, $x \in (A \setminus B) \cup (C \setminus B)$

so $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

since the two sets are contained in each other we conclu[de]

$(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$

universal quantifier

informal symbol

$\boxed{\text{for all}}$, for every, for each $\forall$

Existential quantifier

$\boxed{\text{There exists}}$, for some, for one $\exists$

Examples:

Definition of continuity

- for all $\varepsilon > 0$, there exists $\delta_\varepsilon > 0$ such that, for all $x, c \in \mathbb{R}$ w/ c fixed, if $|x - c| < \delta$, then $|f(x) - f(c)| < \varepsilon$.
- for all students in math131 this summer, there exists an instructor running the course

Odd & even integers

let a & b be even integers

prove that $a+b$, $a-b$, ab are also even integers

odd

an integer n is odd if there exists some integer k such that $n = 2k+1$

even

An integer n is even if there exists some integer k such that $n = 2k$

Proof: since a & b are even integers, there exist integers k & l such that $a = 2k$ & $b = 2l$. So we have

$$a+b = 2k+2l = 2(k+l),$$
$$a-b = 2k-2l = 2(k-l),$$
$$ab = (2k)(2l) = 4kl = 2(2kl)$$

since $k+l$, $k-l$, $2kl$ are also integers, we conclude that $a+b$, $a-b$ & ab is even

Use a counterexample to show that $\frac{a}{b}$ w/ $b \neq 0$ does not have to be an even integer

counterexample: let $a = 6$ & $b = 2$. Then a & b are even but $\frac{a}{b} = \frac{6}{2} = 3$, which is odd

so $\frac{a}{b}$ is not an even integer

Ex.

let a & b be even integers. Prove that $a+b+3$ is an odd integer

we have

$$\begin{aligned} a+b+3 &= 2k+2l+3 \\ &= 2k+2l+2+1 \\ &= 2(k+l+1)+1 \end{aligned}$$

since $k+l+1$ is also an integer we conclude that $a+b+3$ is odd

Discussion 6/26/19 continued

let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$f(x) = x^2 - 4x + 4$

prove that $f(x) \geq 0$ for all $x \in \mathbb{R}$

Proof: for all $x \in \mathbb{R}$, we have

$$
\begin{aligned}
f(x) &= x^2 - 4x + 4 \\
&= (x-2)^2 \\
&\geq 0
\end{aligned}
$$

$f(x) = x^2 - 2x + 2$

prove that $f(x) > 0$ for all $x \in \mathbb{R}$

Proof: for all $x \in \mathbb{R}$ we have

$$
\begin{aligned}
f(x) &= x^2 - 2x + 2 \\
\text{complete square} &= x^2 - 2x + 1 + 1 \\
&= (x-1)^2 + 1 \\
&\geq 0 + 1 \\
&= 1 \\
&> 0
\end{aligned}
$$