Discussion 01

06-26-19

Set Containment Proofs

Example 1: Prove $(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$

Let A, B be subjects of $\mathbb{R}^n$

UNION: If $x \in A \cup B$, then $x \in A$ or $x \in B$

INTERSECTION: If $x \in A \cap B$, then $x \in A$ and $x \in B$

If $x \in A$ implies $x \in B$, then $A \subset B$
A is contained in B

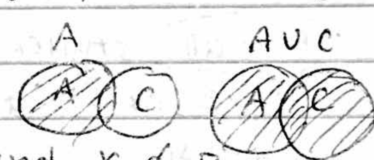
If $A \subset B$ and $B \subset A$, the $A = B$

$A \setminus B = \{x \in A : x \notin B\}$

Proof: First, we will prove $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$

Let $x \in (A \setminus B) \cup (C \setminus B)$. Then $x \in A \setminus B$ or $x \in C \setminus B$

If $x \in A \setminus B$, then $x \in A$ and $x \notin B$. Since $A \subset A \cup C$, we have $x \in A \cup C$ and $x \notin B$.



If $x \in C \setminus B$, then $x \in C$ and $x \notin B$. Since $C \subset A \cup C$, we have $x \in A \cup C$ and $x \notin B$

In either case, $x \in A \cup C$ and $x \notin B$ so $x \in (A \cup C) \setminus B$. Therefore, $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$

Next, we will prove $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

Let, $x \in (A \cup C) \setminus B$
Then $x \in A \cup C$ and $x \notin B$. So we have $x \in A$ and $x \notin B$, or $x \in C$ and $x \notin B$. and so $x \in A$ and $x \notin B$, or $x \in C$ and $x \notin B$.

So $x \in A \setminus B$, or $x \in C \setminus B$

Therefor, $x \in (A \setminus B) \cup (C \setminus B)$

So $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$.

Since the two sets are contained in each other, we conclude

$(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$

QUIZ # 1 MATERIAL

<u>universal quantifier</u>

$\boxed{\text{FOR ALL}}$, for every, for each

informal symbol

$\forall$

<u>Existential quantifier</u>

$\boxed{\text{There exists}}$, for some, for one

informal symbol

$\exists$

<u>Examples</u>

Definition of continuity

- For all $\varepsilon > 0$, there exists $\delta_\varepsilon > 0$ such that, for all $x, c \in \mathbb{R}$, with c fixed if $|x - c| < \delta$, then $|f(x) - f(c)| < \varepsilon$
- For all students in MATH 131 the summer, there exists an instructor running the course
- For all TV programs on NBC, there exists the Ellen Degeneres show

<u>Odd and even integers</u>

Let a and b be even integers
Prove that $a + b$, $a - b$, ab are also even integers

<u>odd</u> : An integer n is odd if there exists some integer k such that
$n = 2k + 1$

<u>even</u> : An integer n is even if there exists some integer k such that $n = 2k$

Proof: Since a and b are even integers, there exist integers k and ℓ such that
$a = 2k$ and 2ℓ. So we have

$a+b = 2k + 2\ell = 2(k+\ell)$.
$a-b = 2k + 2\ell = 2(k-\ell)$.
$ab = (2k)(2\ell) = 4k\ell = 2(2k\ell)$

Since $k+\ell$, $k-\ell$, and $2k\ell$ are also integers, we conclude that $a+b$, $a-b$, and ab is even

Use a counterexample to show that $\frac{a}{b}$ with $b \neq 0$ does not have to be ~~an integer~~

COUNTEREXAMPLE

Let $a = 6$ & $b = 2$ Then a and b are even

But $\frac{a}{b} = \frac{6}{2} = 3$, which is odd.

So $\frac{a}{b}$ is not necessarily an even integer

Odd and even integers

Let a and b be even integers. Prove that $a+b+3$ is an odd integer

~~We have~~

$$
\begin{aligned}
a+b+3 &= 2k + 2\ell + 3 \\
&= 2k + 2\ell + 2 + 1 \\
&= 2(k + \ell + 1) + 1
\end{aligned}
$$

Since $k+\ell+1$ is also an integer we conclude that $a+b+3$ is odd

Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^2 - 4x + 4$$

Prove that $f(x) \geq 0$ for all $x \in \mathbb{R}$

Proof: For all $x \in \mathbb{R}$, we have

$$x^2 - 4x + 4 = (x-2)^2 \geq 0$$

$$f(x) = x^2 - 2x + 2$$

complete the square

$$\begin{aligned} &= x^2 - 2x + 1 + 1 \\ &= (x-1)^2 + 1 \\ &\geq 0 + 1 \\ &= 1 > 0 \end{aligned}$$