

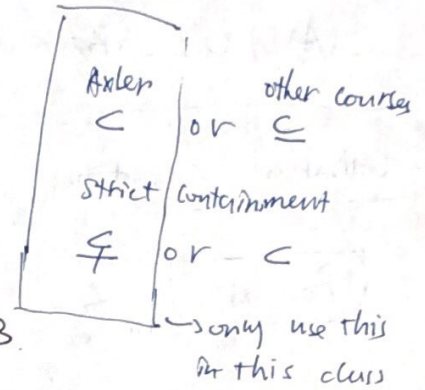
Discussion #1

Example 1: Prove $(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$

Let A, B be subset of \mathbb{R}^n .

Union: If $x \in A \cup B$, then $x \in A$ or $x \in B$

Intersection: If $x \in A \cap B$, then $x \in A$ and $x \in B$.



If $x \in A$ implies $x \in B$, then $A \subset B$. A is contained in B .

If $A \subset B$ and $B \subset A$, then $A = B$.

Proof:

$A \setminus B = \{x \in A : x \notin B\}$ → read as "A minus B".

First, we will prove $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$ Forward containment

Let $x \in (A \setminus B) \cup (C \setminus B)$. Then $x \in A \setminus B$ or $x \in C \setminus B$.

If $x \in A \setminus B$, then $x \in A$ and $x \notin B$. Since $A \subset A \cup C$, we have $x \in A \cup C$ and $x \notin B$.

If $x \in C \setminus B$, then $x \in C$ and $x \notin B$. Since $C \subset A \cup C$, we have $x \in A \cup C$ and $x \notin B$.

In either case, we have $x \in A \cup C$ and $x \notin B$. So $x \in (A \cup C) \setminus B$.

Therefore, $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$.

Next, we will prove $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$ Backward containment

Let $x \in (A \cup C) \setminus B$. Then $x \in A \cup C$ and $x \notin B$.

So we have $x \in A$ and $x \notin B$ or $x \in C$ and $x \notin B$.

and so $x \in A$ and $x \notin B$, or $x \in C$ and $x \notin B$

So $x \in A \setminus B$, or $x \in C \setminus B$. Therefore, $x \in (A \setminus B) \cup (C \setminus B)$.

So $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

Since the two sets are contained in each other, we conclude

$$(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B.$$

Universal quantifier

For all, for every, for each

informal symbol
 \forall

Existential quantifier

There exists, for some, for one \exists

Ex: Definition of continuity

- For all $\epsilon > 0$, there exists $\delta > 0$ such that, for all $x, c \in \mathbb{R}$ with c fixed, if $|x - c| < \delta$, then $|f(x) - f(c)| < \epsilon$.
- For all students in MATH 131 this summer, there exists an instructor running the course.
- For all TV programs on NBC, there exists the Ellen Degeneres show.

Odd and even integers

Let ~~a~~ a and b be even integers.

Prove that $a+b$, $a-b$, ab are also even integers.

Odd An integer n is odd if there exists some integer k such that $n = 2k+1$

Even An integer n is even if there exists some integer k such that $n = 2k$

Proof: Since a and b are even integers, there exist integers k and l such that $a = 2k$ and $b = 2l$. So we have

$$a+b = 2k+2l = 2(k+l),$$

$$a-b = 2k-2l = 2(k-l),$$

$$ab = (2k)(2l) = 4kl = 2(2kl).$$

Since $k+l$, $k-l$, and $2kl$ are also integers, we conclude that $a+b$, $a-b$, and ab is even.

Use a counter example to show that $\frac{a}{b}$ with $b \neq 0$ does not have to be an even integer.

Counter Example:

Let $a = 6$ and $b = 2$. Then a and b are even.

But $\frac{a}{b} = \frac{6}{2} = 3$, which is odd.

So $\frac{a}{b}$ is not necessarily an even integer.

Odd and Even integers

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Let a and b be even integers.

Prove that $a+b+3$ is an odd integer.

$$\begin{aligned}\text{We have } a+b+3 &= 2k+2l+3 \\ &= 2k+2l+2+1 \\ &= 2(k+l+1)+1\end{aligned}$$

Since $k+l+1$ is also an integer, we conclude that $a+b+3$ is odd.

Ex:

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 4x + 4$.

Prove that $f(x) \geq 0$ for all $x \in \mathbb{R}$

Proof: For all $x \in \mathbb{R}$, we have

$$f(x) = x^2 - 4x + 4 = (x-2)^2 \geq 0$$

Ex: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 2x + 2$

Prove that $f(x) > 0$ for all $x \in \mathbb{R}$

Proof: For all $x \in \mathbb{R}$ we have

$$f(x) = x^2 - 2x + 2 = x^2 - 2x + 1 + 1 = (x-1)^2 + 1 \geq 0 + 1 = 1 > 0$$

Complete square