Discussion #1

Example 1: Prove $(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$

Let A, B be subject of $\mathbb{R}^n$.

Union: If $x \in A \cup B$, then $x \in A$ or $x \in B$

Intersection: If $x \in A \cap B$, then $x \in A$ and $x \in B$.

Axler	other courses
$\subset$	or $\subseteq$
strict	containment
$\subsetneq$	or $\subset$

→ only use this in this class

If $x \in A$ implies $x \in B$, then $A \subset B$. A is contained in B.

If $A \subset B$ and $B \subset A$, then $A = B$.

proof:

$A \setminus B = \{x \in A : x \notin B\}$ → read as "A minus B".

First, we will prove $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$ Forward containment

Let $x \in (A \setminus B) \cup (C \setminus B)$. Then $x \in A \setminus B$ or $x \in C \setminus B$.

If $x \in A \setminus B$, then $x \in A$ and $x \notin B$. Since $A \subset A \cup C$, we have $x \in A \cup C$ and $x \notin B$.

If $x \in C \setminus B$, then $x \in C$ and $x \notin B$. Since $C \subset A \cup C$, we have $x \in A \cup C$ and $x \notin B$.

In either case, we have $x \in A \cup C$ and $x \notin B$. So $x \in (A \cup C) \setminus B$.

Therefore, $(A \setminus B) \cup (C \setminus B) \subset (A \cup C) \setminus B$.

Next, we will prove $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$ Backward containment

Let $x \in (A \cup C) \setminus B$. Then $x \in A \cup C$ and $x \notin B$.

So we have $x \in A$ ~~and $x \notin B$~~ or $x \in C$ and $x \notin B$.

and so $x \in A$ and $x \notin B$, or $x \in C$ and $x \notin B$

So $x \in A \setminus B$, or $x \in C \setminus B$. Therefore, $x \in (A \setminus B) \cup (C \setminus B)$.

So $(A \cup C) \setminus B \subset (A \setminus B) \cup (C \setminus B)$

Since the two sets are contained in each other, we conclude

$(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$.

Universal quantifier

$\boxed{\text{For all}}$, for every, for each

informal symbol

$\forall$

Existential quantifier

$\boxed{\text{There exists}}$, for some, for one

$\exists$

Ex: Definition of continuity

- $\boxed{\text{For all}}$ $\varepsilon > 0$, $\boxed{\text{there exists}}$ $\delta_\varepsilon > 0$ such that, for all $x, c \in \mathbb{R}$ with c fixed, if $|x-c| < \delta$, then $|f(x)-f(c)| < \varepsilon$.
- $\boxed{\text{For all}}$ students in MATH 131 this sumer, $\boxed{\text{there exists}}$ an instructor running the course.
- $\boxed{\text{For all}}$ TV programs on NBC, $\boxed{\text{there exists}}$ the Ellen Degeneres Show.

Odd and even integers

Let a and b be even integers.

Prove that $a+b$, $a-b$, ab are also even integers.

<u>Odd</u> An integer n is <u>odd</u> if <u>there exists</u> some integer k such that $n = 2k+1$

<u>Even</u> An integer n is <u>even</u> if <u>there exists</u> some integer k such that $n = 2k$

<u>Proof</u>: Since a and b are even integers, there exist, integers k and l such that $a = 2k$ and $b = 2l$. So we have

$$a+b = 2k+2l = 2(k+l),$$
$$a-b = 2k-2l = 2(k-l),$$
$$ab = (2k)(2l) = 4kl = 2(2kl).$$

Since $k+l$, $k-l$, and $2kl$ are also integers, we conclude that $a+b$, $a-b$, and ab is even.

Use a counter example to show that $\frac{a}{b}$ with $b \neq 0$ does not have to be an even integer.

<u>Counter Example:</u>

Let $a = 6$ and $b = 2$. Then a and b are even.

But $\frac{a}{b} = \frac{6}{2} = 3$, which is odd.

So $\frac{a}{b}$ is not necessarily an even integer.

Odd and Even integers

~~Yongxian Wu~~

Let a and b be even integers.

Prove that $a+b+3$ is an odd integer.

We have
$$\begin{aligned} a+b+3 &= 2k+2l+3 \\ &= 2k+2l+2+1 \\ &= 2(k+l+1)+1 \end{aligned}$$

Since $k+l+1$ is also an integer, we conclude that $a+b+3$ is odd.

Ex:

Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = x^2-4x+4$.

Prove that $f(x) \geq 0$ for all $x \in \mathbb{R}$

Proof: For all $x \in \mathbb{R}$, we have
$$f(x) = x^2-4x+4 = (x-2)^2 \geq 0$$

Ex: Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = x^2-2x+2$

Prove that $f(x) > 0$ for all $x \in \mathbb{R}$

Proof: For all $x \in \mathbb{R}$ we have
$$f(x) = x^2-2x+2 = x^2-2x+1+1 = (x-1)^2+1 \geq 0+1 = 1 > 0$$

Complete square