MATH 131: Linear Algebra I

University of California, Riverside
Group Examination 1
Time limit: 60 minutes
Score: _____ / 100
June 27, 2019

This group examination is open textbook, open lecture notes, open homework, and open classmates.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:______________________________ Student ID:______________________________

(20pts) 1. For this question, you will need to refer to the definitions in Chapter 1 of your Axler textbook to find the answers. (This is one of the reasons this examination is open book, open notes, open homework, and open classmates.)

(4pts) a. Write down the definitions of *addition* and *scalar multiplication* on a set V.

(12pts) b. Now assume that V is a *vector space* over a field $\mathbb{F}$. Write down all the properties of a vector space.

(4pts) c. Write down the definition of a *subspace* U of V.

(20pts) 2. Let n be a positive integer, let $\lambda \in \mathbb{F}$ be a scalar, and let $x, y \in \mathbb{F}^n$ be lists of length n. Define on $\mathbb{F}^n$ the operations of "addition"

$$x \text{ ``+'' } y = x - y$$

and "scalar multiplication"

$$\lambda \text{ ``}\times\text{'' } x = -\lambda x.$$

Is $\mathbb{F}^n$ a vector space over $\mathbb{F}$ with respect to these operations? If so, prove it. If not, prove which of the properties of a vector space are satisfied and give counterexamples for the properties of a vector space that are not satisfied.

(20pts) 3. For each of the following subsets of $\mathbb{F}^3$, determine whether it is a subspace of $\mathbb{F}^3$. If so, prove it. If not, give a counterexample to show some property of a subspace that is not satisfied.

(5pts) a. $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 + 2x_2 + 3x_3 = 0\}$;

(5pts) b. $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 + 2x_2 + 3x_3 = 4\}$;

(5pts) c. $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 x_2 x_3 = 0\}$;

(5pts) d. $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 = 5x_3\}$.

(20pts) 4. Let $\mathbb{R}^{\mathbb{R}}$ be the set of all real-valued functions $f : \mathbb{R} \to \mathbb{R}$. A real-valued function $f : \mathbb{R} \to \mathbb{R}$ is called *even* if

$$f(-x) = f(x)$$

for all $x \in \mathbb{R}$. A real-valued function $f : \mathbb{R} \to \mathbb{R}$ is called *odd* if

$$f(-x) = -f(x)$$

for all $x \in \mathbb{R}$. Let U_e denote the set of real-valued even functions on $\mathbb{R}$, and let U_o denote the set of real-valued odd functions on $\mathbb{R}$. Show that we have $\mathbb{R}^{\mathbb{R}} = U_e \oplus U_o$.

(20pts) 5. Let $\mathbb{C}^{\mathbb{R}}$ be the set of all complex-valued functions $f : \mathbb{R} \to \mathbb{C}$. Define on $\mathbb{C}^{\mathbb{R}}$ the usual operations of addition

$$(f + g)(x) = f(x) + g(x)$$

and scalar multiplication

$$(\lambda f)(x) = \lambda f(x)$$

for all scalars $\lambda \in \mathbb{F}$ and complex-valued functions $f, g \in \mathbb{C}^{\mathbb{R}}$. Also let U be the set of all complex-valued functions $f : \mathbb{R} \to \mathbb{C}$ such that

$$f(-x) = \overline{f(x)}$$

for all $x \in \mathbb{R}$, where the bar denotes the complex conjugate.

(12pts) a. Show that $\mathbb{C}^{\mathbb{R}}$ is a vector space over $\mathbb{R}$ with the operations defined above.

(8pts) b. Show that U is a subspace of $\mathbb{C}^{\mathbb{R}}$.