MATH 131: Linear Algebra I

University of California, Riverside

Group Examination 2

Time limit: 60 minutes

Score: _____ / 100

July 3, 2019

This group examination is open textbook, open lecture notes, open homework, and open classmates.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:______________________________ Student ID:______________________________

(20pts) 1. For this question, you will need to refer to the definitions in Chapter 2 of your Axler textbook to find the answers.

(4pts) a. Write down the definitions of *linear combination* and *span*.

(4pts) b. Write down the definitions of *linearly independent* and *linearly dependent*.

(4pts) c. Write down the definitions of *finite-dimensional vector space* and *infinite-dimensional vector space*.

(4pts) d. Write down the definitions of *polynomial* and *degree of a polynomial*.

(4pts) e. Write down the definitions of *basis* and *dimension*.

(20pts) 2. Suppose $v_1, v_2, v_3, \ldots, v_m$ is a linearly independent list of vectors in the vector space V.

(8pts) a. Prove that $5v_1 - 4v_2, v_2, v_3, \ldots, v_m$ is linearly independent.

(8pts) b. If $\lambda \in \mathbb{F}$ satisfies $\lambda \neq 0$, prove that $\lambda v_1, \lambda v_2, \lambda v_3, \ldots, \lambda v_m$ is linearly independent.

(4pts) c. Assume that $w_1, \ldots, w_m$ is also a linearly independent list of vectors in V. Give a counterexample to show that the list $v_1 + w_1, \ldots, v_m + w_m$ is not linearly independent.

(20pts) 3. Consider the vector space $\mathbb{F}^3$ with the standard basis $(1, 0, 0), (0, 1, 0), (0, 0, 1)$.

(15pts) a. Show that the list $(1, 0, -1), (1, 2, 1), (0, -3, 2)$ is a basis of $\mathbb{F}^3$.

(5pts) b. Express the standard basis vectors $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ as a linear combination of the list in part (a).

(20pts) 4. Let V be a vector space. Suppose v_1, v_2, v_3, v_4 is a basis of V. Prove that the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$

is also a basis of V.

(20pts) 5. Consider the sets

$$U_1 = \{(x_1, x_2, x_3, x_4) \in \mathbb{F}^4 : x_1 + x_2 = 0\}$$

and

$$U_2 = \{(x_1, x_2, x_3, x_4) \in \mathbb{F}^4 : x_1 + x_3 = 0\}.$$

(6pts) a. Show that U_1 and U_2 are subspaces of $\mathbb{F}^4$.

(12pts) b. Find the dimensions of U_1, U_2, $U_1 + U_2$, and $U_1 \cap U_2$. For this part of the question, do NOT use the formula provided by 2.43 of Axler to find the dimension of $U_1 + U_2$; any justification using that formula will receive an automatic zero score for part (b).

Hints: Find a basis of each of the three sets, and prove that they are indeed bases of their respective sets. What is the length of each basis?

(2pts) c. Write down the formula from 2.43 of Axler that represents the dimension of $U_1 + U_2$. Substitute the values you obtained in part (b) into the formula to verify that it holds true for the dimensions of our subspaces U_1, U_2, $U_1 + U_2$, and $U_1 \cap U_2$.