MATH 131: Linear Algebra I

University of California, Riverside
Group Examination 3
Time limit: 60 minutes
Score: _____ / 100
July 11, 2019

This group examination is open textbook, open lecture notes, open homework, and open classmates.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:_____________________________ Student ID:_____________________________

(20pts) 1. For this question, you will need to refer to the definitions in Chapter 3, Sections A-B, of your Axler textbook to find the answers.

(4pts) a. Write down the properties of a *linear map* from V to W.

(4pts) b. Write down the differentiation and integration maps, and prove that they are linear.

(4pts) c. Write down the definitions of *addition* and *scalar multiplication* on $\mathcal{L}(V, W)$.

(4pts) d. Write down the definitions of *null space* and *range*.

(4pts) e. Write down the definitions of *injective* and *surjective*.

(20pts) 2. Which of the following maps $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear? If the map is linear, prove it. If not, give a counterexample to show that some property of a linear map that is not satisfied.

(4pts) a. $T(x_1, x_2) = (x_1 + 1, x_2)$

(4pts) b. $T(x_1, x_2) = (x_2, x_1)$

(4pts) c. $T(x_1, x_2) = (|x_1|, x_2)$

(4pts) d. $T(x_1, x_2) = (\sin x_1, x_2)$

(4pts) e. $T(x_1, x_2) = (x_1 - x_2, 0)$

(20pts) 3. Consider a linear map $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, -1, 1) = (1, 0)$ and $T(1, 1, 1) = (0, 1)$.

(12pts) a. Show that there exists such a map.

Hint: First verify that $(1, -1, 1), (1, 1, 1)$ is a linearly independent list of vectors in $\mathbb{R}^3$. Then use 2.33 of Axler to show that this linearly independent list extends to a basis of $\mathbb{R}^3$. Finally, use 3.5 of Axler to arrive at your desired conclusion.

(8pts) b. Compute $T(-5, 1, -5)$.

(20pts) 4. Suppose V and W are both finite-dimensional. Prove that there exists an injective map $T \in \mathcal{L}(V, W)$ if and only if $\dim V \leq \dim W$.

(20pts) 5. Suppose V and W are both finite-dimensional. Prove that there exists a surjective map $T \in \mathcal{L}(V, W)$ if and only if $\dim V \geq \dim W$.