Ex)
$\lambda = 2-i \quad x \in \mathbb{C}^2$ ST $x = (3-2i, 5i)$

Find $\lambda x = (2-i)(3-2i, 5i)$
$= (6-3i-4i-2, 10i+5)$
$= (4-7i, 5+10i)$

Check 1.A
4, 5, 6, 9, 10, 11, 12, 13, 15, 16

6/25 1.B Vector Space $(V, +, \cdot)$
set of Vectors, add, Scalar multiplication

Def) Vector Space consists of a set of Vectors and a field F.
- vectors can be added to create another vector.
  $\forall \vec{u}, \vec{v} \in V, \vec{u} + \vec{v} \in V$
- Scalar can be multiplied with a vector to a new vector as well.

★ Add & Scalar mult. must satisfy the following:

1. commutativity: $\vec{u} + \vec{v} = \vec{v} + \vec{u} \quad \forall \vec{u}, \vec{v} \in V$
2. Associativity: $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w}) \quad \forall \vec{u}, \vec{v}, \vec{w} \in V$
   $(\alpha B)\vec{v} = \alpha(B\vec{v}) \quad \forall \alpha, B \in F$
3. Add Identity $\exists \vec{0} \in V$ ST $\vec{0} + \vec{v} = \vec{v} \quad \forall \vec{v} = V$
4. dd Inverse $\forall \vec{v} \in V, \exists \vec{w} \in V$ ST $\vec{v} + \vec{w} = \vec{0}$
   note: $(\vec{w} = -\vec{v})$
5. multi. identity $1\vec{v} = \vec{v} \quad \forall \vec{v} \in V$
6. Distributive prop. $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v}$
   $(\alpha + B)\vec{v} = \alpha\vec{v} + B\vec{v}$
   $\forall \vec{u}, \vec{v} \in V$
   $\alpha, B \in F$

$V = F^S = \{ : S \to F \}$ = set of functions from set S to field F

add: $f, g = F^S$
$(f+g)(x) = f(x) + g(x) \quad \forall x \in S$

scalar multi.: $(af)(x) = a f(x) \quad \forall a \in F$
$\forall f \in F^S$

F^S is a vector space over F

☆ Properties of Vector Spaces:

1. vector space has a <u>unique</u> add Identity

pf: $0 + 0' = 0'$
$\| $ commutative
$0' + 0 = 0'$
$\| $ add identity
$0 \quad 0' = 0 \vec{v}$

2. unique add Inverse

pf: $\exists\, w, w' \in V$ st
$v + w = 0$
$v + w' = 0$
$w + w = 0 = w + (v + w') = \underbrace{(w + v)}_{0} + w' = w'$
$w = w'$

Note

- $-\vec{v} \to$ add. inverse of $\vec{v}$
- $\vec{w} - \vec{v} = \vec{w} + (-\vec{v})$

3. 0 times a vector is always a vector
$0\vec{v} = \vec{0}$

Pf: $0\vec{v} = (0+0)\vec{v} = 0\vec{v} + 0\vec{v} = \boxed{0\vec{v} = 0}$
Distributive

4. $a\vec{0} = \vec{0} \quad \forall a \in F$

Pf: $a\vec{0} = a(\vec{0} + \vec{0}) = a\vec{0} + a\vec{0} = a\vec{0} = \vec{0}$

5. (-1) times a vector

$(-1)\vec{v} = -v$ for any $\vec{v} \in V$

$\vec{v} + \vec{w} = 0$ unique

$w = -v$

$v + (-v) = 0$

$\vec{v} + (-1)v = 0$?

$1\vec{v} + (-1)\vec{v} = (1 + (-1))\vec{v}$

$= 0\vec{v} = \vec{0}$ ✓

$a\vec{v} + B\vec{v} = (a+B)\vec{v}$

$\quad = 1 \quad = -1$

Exercise 1.B

2. suppose $a \in F$, vector $\in V$ and $a\vec{v} = \vec{0}$

prove that $a = 0$ or $\vec{v} = \vec{0}$

given $a\vec{v} = \vec{0}$

prop 3 states $0 \cdot \vec{v} = \vec{0}$

a could be 0

or $\vec{v}$ could be $\vec{0}$ and $a\vec{0} = \vec{0}$

6/26 1.C Subspaces

1.32 Def

Let V = vector space

Let U = subset of V

U = subspace of V IFF U is a vector space with add. & multi. scalar

1.34 Conditions of subspace

1. Add. Identity $0 \in U$
2. Closed under add. : $u, w \in U$ implies $u + w \in U$ ☆
3. Closed under multi : $a \in F$ and $u \in U$ implies $au \in U$.

Ex) 1.35 a) If $b \in F$, then $\{(x_1, x_2, x_3, x_4) \in F^4 : x_3 = 5x_4 + b\}$ is a subspace if $b = 0$

Add Identi. Yes $0 = 5(0) + b = \boxed{0 = b}$