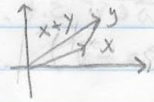
Geometric Intuition:

$x, y \in \mathbb{R}^2$



Def Scalar multiplication

$\lambda \in F, (x_1, x_2, \ldots, x_n) \in F^n$

$\lambda(x_1, x_2, \ldots, x_n) := (\lambda x_1, \lambda x_2, \ldots, \lambda x_n)$

Ex. $\lambda = 2 - i$, $x \in \mathbb{C}^2$ s.t. $x = (3-2i, 5i)$

Find $\lambda x = (2-i)(3-2i, 5i)$

$= ((2-i)(3-2i), (2-i)(5i))$

$= (6 - 4i - 3i + 2i^2, 10i - 5i^2)$

$= (6 - 2 - 7i, 10i + 5) = \boxed{(4-7i, 5+10i)}$

Ex. 1.A: 4, 5, 6, 9, 10, 11, 12, 13, 15, 16

6/25/19 1.B vector space
week 1
Tues.

$(V, +, \cdot)$

V ↓ set of vectors; $+$ ↓ addition; $\cdot$ ↳ scalar multiplication

A vector space consists of a set of vectors V & a field F

- the vectors can be added to yield another vector in V:

  $\forall \vec{u}, \vec{v} \in V$, $\vec{u} + \vec{v} \in V$

- the scalar can be multiplied w/ the vector to yield a new vector in V:

  If $\alpha \in F$, $\vec{v} \in V$ then $\alpha \cdot \vec{v} \in V$

Addition & scalar multiplication must also satisfy the following axioms:

1) commutivity $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ $\forall \vec{u}, \vec{v} \in V$

2) associativity $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ $\forall \vec{u}, \vec{v}, \vec{w} \in V$

$(\alpha \beta)\vec{v} = \alpha(\beta \vec{v})$ $\forall \alpha, \beta \in F$

3) additive identity $\exists\, \vec{0} \in V$ s.t. $\vec{0} + \vec{v} = \vec{v} \quad \forall\, \vec{v} \in V$

4) additive inverse, $\forall\, \vec{v} \in V$, $\exists\, \vec{w} \in V$ s.t. $\vec{v} + \vec{w} = \vec{0}$

(Notation: $\vec{w} = -\vec{v}$)

5) Multiplication identity

$1\vec{v} = \vec{v} \quad \forall\, \vec{v} \in V$

6) Distributive Property

$\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v} \quad \forall\, \vec{u}, \vec{v} \in V$

$(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v} \quad \alpha, \beta \in F$

Remarks:

1) Elements of a vectorspace are called vector

2) we will say that V is a vectorspace over F instead of saying simply the V is a vectorspace

Ex: $\mathbb{R}, \mathbb{R}^2, \ldots, \mathbb{R}^n$ are vectorspaces

$\mathbb{C}, \mathbb{C}^2, \ldots, \mathbb{C}^n$ " " "

Def.

- A vector space over $\mathbb{R}$ (i.e. $F = \mathbb{R}$) is called a real vectorspa
- A vectorspace over $\mathbb{C}$ (i.e $F = \mathbb{C}$) is called a complex vectorsp

Ex. Recall; $F = \mathbb{R}$ or $\mathbb{C}$

$V = F^n := \{(x_1, x_2, \ldots, x_n) \mid x_1, x_2, \ldots, x_n \in F\}$

Addition: $\underbrace{(x_1, x_2, \ldots, x_n)}_{F^n} + \underbrace{(y_1, y_2, \ldots, y_n)}_{\in F^n} := \underbrace{(x_1+y_1, x_2+y_2, \ldots, x_n+}_{\in F^n}$

scalar multiplication:

$\alpha \in F$

$\alpha(x_1, x_2, \ldots, x_n) := (\alpha x_1, \alpha x_2, \ldots, \alpha x_n) \in F^n$

with this Def $(F^n, +, \cdot)$ becomes a vector space over F

Ex. $\mathbb{R}^3 = \{(x_1, x_2, x_3) \mid x_i \in \mathbb{R}\}$ over $\mathbb{R}$.

$F = \mathbb{R}$ $\quad +: \underbrace{(x_1 + x_2, x_3)}_{\substack{\text{vector} \\ \text{in } \mathbb{R}^3}} + (y_1, y_2, y_3) = (x_1+y_1, x_2+y_2, x_3+y_3)$

$n = 3$

$\cdot : \underset{\text{scalar}}{\alpha}(x_1, x_2, x_3) := \underbrace{(\alpha x_1, \alpha x_2, \alpha x_3)}_{\text{vector}} \in \mathbb{R}^3$

Ex.

$V = F^S = \{f : S \to F\}$ = set of functions from set S to field F

- Addition, $f, g \in F^S$

  $(f+g)(x) = f(x) + g(x) \quad \forall x \in S$

- scalar multiplication $(\alpha f)(x) = \alpha f(x) \quad \forall \alpha \in F$, $\forall f \in F^S$

Properties vector space:

I) A vectorspace has a unique additive identity

Recall: additive Id: $0 + \vec{v} = \vec{v}$

Proof

suppose there are two additive Ids: 0 & $0'$

$\vec{0} + \vec{v} = \vec{v}$ & $\vec{0'} + \vec{v} = \vec{v}$

$0 + v = v$ → 0 is additive id.

$0 + 0' = 0'$

$\parallel$ commutivity

$0' + 0$

$\parallel$ → is also additive id.

$0 \Rightarrow 0' = 0$ ✓

II) unique additive inverse

proof

V vectorspace $\vec{v} \in V$, suppose there are two additive inverse

ie $\exists w, w' \in V$ s.t.

$v + w = 0$

$v + w' = 0$ $\overset{WTS}{\Longrightarrow} w = w'$?

$w = w + 0 = w + (v + w') = \underbrace{(w + v)}_{0} + w' = w'$

$\Rightarrow \boxed{w = w'}$

Notation:

- $-\vec{v}$ → additive inverse of $\vec{v}$
- $\vec{w} - \vec{v} = \vec{w} + (-\vec{v})$

III) the # 0 times a vector

$0\vec{v} = \vec{0}$

scalar, vector, vector (additive identity)

Proof

$0\vec{v} = (0+0)\vec{v} = 0\vec{v} + 0\vec{v} \Rightarrow \boxed{0\vec{v} = 0}$

distributive property

$(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$

IV) $\alpha\vec{0} = \vec{0} \quad \forall \alpha \in F$

proof: $\cancel{\alpha\vec{0}} = \alpha(\vec{0}+\vec{0}) = \cancel{\alpha\vec{0}} + \alpha\vec{0} \Rightarrow \boxed{\alpha\vec{0} = \vec{0}}$

V) the # (-1) times a vector

$(-1)\vec{v} = -\vec{v}$ for any $\vec{v} \in V$

$\vec{v} + \vec{w} = 0$ unique

$w = -v$

$v + (-v) = 0$

$\vec{v} + (-1)\vec{v} = 0$?

use "$\alpha\vec{v} + \beta\vec{v} = (\alpha+\beta)\vec{v}$"

$(1 + (-1))\vec{v}$

$= 0\vec{v} = \vec{0}$ ✓

Ex 1.B

2) suppose $a \in F$, $v \in V$ & $a\vec{v} = \vec{0}$. prove that $a = 0$ or $\vec{v} = \vec{0}$

If $a = 0$ ✓

what if $a \neq 0 \overset{?}{\Rightarrow} \vec{v} = \vec{0}$

$a \neq 0$ so $\frac{1}{a}$ is well defined

$\frac{1}{a}(a\vec{v}) = (\frac{1}{a}\vec{0})$

$(\frac{1}{a}a)\vec{v} = \vec{0}$

$\boxed{\vec{v} = \vec{0}}$ ✓

6/26/19

week 1 Wednesday

Email: ryanta@math.ucr.edu

OH: MTWR 11:10 AM - 12:00 PM at CHASS 2134

Quiz 1 in discussion today

Direct Proofs

3:15 pm - 4:00 pm

open book, open notes

Group exam 1 tomorrow

- 20 pts on definitions
- 40 pts on 2 Axler exercises
  - look at: Axler exercises 1.C.1 & 1.C.24
- 40 pts on curveballs
  - know & apply properties of vector space & subspace

1.C Subspaces

let V be a vector space (w/ addition & scalar multiplicati

1.32 Definition: Let U be a subset of V. Then U is a $\boxed{\text{subspace}}$ of V if U is also a vector space w/ the same addition & scalar multiplication

1.34 Conditions of subspace

U is a subspace of V if & only if it satisfies:

Additive Identity: $0 \in U$

closed under addition: $u, w \in U$ implies $u + w \in U$

scalar multiplication: $a \in F$ & $u \in U$ implies $au \in U$