MATH 131: Linear Algebra I

University of California, Riverside

Quiz 1

Time limit: 45 minutes

Score: _____ / 50

June 26, 2019

This quiz is open textbook and open lecture notes.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:______________________________ Student ID:______________________________

(10pts) 1. This question will ask you to recall your knowledge of quantifiers that we covered in our discussion today.

(4pts) a. Write the phrases and the informal symbols of both the universal quantifier and the existential quantifier.

(6pts) b. Write 6 different sentences of your own that incorporate both the universal and existential quantifiers in the same sentence. (Do not repeat any of the examples we have already done together in discussion before this quiz; those sentences will give you no credit.)

(10pts) 2. Let a, b be even integers.

(6pts) a. Prove that $a + b, a - b, ab$ are also even integers. Use a counterexample to show that $\frac{a}{b}$ (assuming $b \neq 0$) does not have to be an even integer.

(4pts) b. Prove that $3a^2 - 4b - 5$ is an odd integer.

(10pts) 3. Consider the map $f : \mathbb{R} \to \mathbb{R}$ defined by the function $f(x) = x^3 - 6x^2 + 9x$.

(5pts) a. Prove that we have $f(x) \geq 0$ for all $x \geq 0$.

(3pts) b. Use a counterexample to show that the statement $f(x) \geq 0$ for all $x \in \mathbb{R}$ is false.

(2pts) c. Define a new function $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = -f(x)$. Is it true that we have $g(x) \leq 0$ for all $x \geq 0$? Either prove that this statement is true or give a counterexample to show that this statement is false.

(10pts) 4. Suppose $U_1, \ldots, U_m$ are subspaces of V. Prove that $U_1 + \cdots + U_m$ is the smallest subspace of V containing $U_1, \ldots, U_m$.

(10pts) 5. Suppose U and W are subspaces of a vector space V. Prove that $U + W$ is a direct sum if and only if $U \cap W = \{0\}$.