MATH 131: Linear Algebra I

University of California, Riverside

Quiz 2

Time limit: 45 minutes

Score: _____ / 50

July 10, 2019

This quiz is open textbook and open lecture notes.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:______________________________ Student ID:______________________________

(10pts) 1. Use contradiction to prove that, if $9n^3 + 7n^2 + 5n$ is even, then n is even.

(10pts) 2. Use contradiction to prove that $\sqrt{3}$ is irrational.

(10pts) 3. Prove that a list $v_1, \ldots, v_n$ is a basis of V if and only if every $v \in V$ can be written uniquely in the form $v = a_1 v_1 + \cdots + a_n v_n$, where $a_1, \ldots, a_n \in \mathbb{F}$.

(10pts) 4. Suppose $v_1, \ldots, v_n$ is a basis of V and $w_1, \ldots, w_n \in W$. Prove that there exists a unique linear map $T : V \to W$ such that $Tv_j = w_j$ for each $j = 1, \ldots, n$.

(10pts) 5. Let V and W be vector spaces. If V is finite-dimensional and $T \in \mathcal{L}(V, W)$, prove that $\operatorname{range} T$ is also finite dimensional and we have $\dim V = \dim \operatorname{null} T + \dim \operatorname{range} T$.