MATH 131: Linear Algebra I

University of California, Riverside

Quiz 3

Time limit: 45 minutes

Score: _____ / 50

July 17, 2019

This quiz is open textbook and open lecture notes.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:______________________________ Student ID:______________________________

(10pts) 1. State the contrapositive, converse, and inverse of the following statements in parts a-c. Do not forget to answer part d as well.

(3pts) a. If a real number is greater than 3, then its square is greater than 9.

(3pts) b. If a quadrilateral is a square, then it is a rectangle.

(3pts) c. If Riverside did not exceed 100°F this month, then I would not have had to pay an expensive bill for the air conditioning in my house.

(1pt) d. What is the key difference between a proving a contrapositive or using a proof by contradiction?

(10pts) 2. Suppose n is a positive integer. Consider the statement: If $n^4 + 10n^2 + 21$ is not a multiple of 32, then n is even.

(1pt) a. State the contrapositive of the statement.

(9pts) b. Prove the contrapositive that you have written.

(10pts) 3. Suppose $T \in \mathcal{L}(V, W)$ and $v \in V$. Suppose $v_1, \ldots, v_n$ is a basis of V and $w_1, \ldots, w_n$ is a basis of W. Prove that we have

$$\mathcal{M}(Tv) = \mathcal{M}(T)\mathcal{M}(v).$$

(10pts) 4. Let V and W be finite-dimensional vector spaces over $\mathbb{F}$. Show that V and W are isomorphic if and only if $\dim V = \dim W$.

(10pts) 5. Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. We will prove that the following are equivalent:

(a) T is invertible;

(b) T is injective;

(c) T is surjective.