MATH 131: Linear Algebra I

University of California, Riverside

Quiz 4

Time limit: 45 minutes

Score: _____ / 50

July 24, 2019

This quiz is open textbook and open lecture notes.

By writing my name and student ID number below, I agree to the following terms:

- I promise not to engage in any form of academic dishonesty. In particular, I will not use any resources other than what is listed above. I understand that any act of cheating may cause me to receive a failing grade in the course and further disciplinary action from the university.
- I will turn my cellular phone off and place it on the desk in front of me. If I do not have a cellular phone, I will notify the instructor before the start of any quiz or examination.
- If I need to use the restroom during any exam or quiz, then I must ask the instructor for permission. I cannot use the restroom for more than 15 minutes, more than once, or while another student is using the restroom. Also, I cannot take anything with me to the restroom. If I violate any of these policies, I understand that the instructor may dismiss me early and will only be graded for the work done.
- I will not open this booklet until the instructor tells the class to do so.

Name:_____ Student ID:_____

(10pts) 1. Use mathematical induction to prove the statement

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

for all positive integers n.

(10pts) 2. Use mathematical induction to prove the statement

$$1^3 + 3^3 + 5^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

for all positive integers n. Use this statement to prove that every perfect cube n^3 can be written as a difference of two squares.

(10pts) 3. Use mathematical induction to prove the statement

$$1 + r + r^2 + r^3 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

for all real numbers r satisfying $r \neq 1$ and for all positive integers n.

(10pts) 4. Suppose U is a subspace of V and $v, w \in V$. Prove that the following are equivalent:

(a) $v - w \in U$;

(b) $v + U = w + U$;

(c) $(v + U) \cap (w + U) \neq \varnothing$.

(10pts) 5. Suppose U is a subspace of V. Consider on the quotient space V/U the operations of addition

$$(v + U) + (w + U) = (v + w) + U$$

and scalar multiplication

$$\lambda(v + U) = (\lambda v) + U$$

for all $v, w \in V$ and $\lambda \in \mathbb{F}$. Prove that V/U is a vector space.