

list all bases you need.

Step 1:

$$S_V = \left\{ e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$S_W = \left\{ f_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, f_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$C_V = \left\{ v_1 = e_1, v_2 = e_3, v_3 = e_2 - 3e_1 \right\}$$

$$C_W = \left\{ w_1 = f_1 - f_2, w_2 = 2f_1 - f_2 \right\}$$

Step 2: Write vectors in C_V in S_V -coordinates.

for V

$$v_1 = e_1, \text{ so } [v_1]_{S_V} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$v_2 = e_3, \text{ so } [v_2]_{S_V} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v_3 = e_2 - 3e_1, \text{ so } [v_3]_{S_V} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

Step 3

for V

Then

$$P_{S_V \leftarrow C_V} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Step 2

for W

$$w_1 = f_1 - f_2, \quad [w_1]_{S_W} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$w_2 = 2f_1 - f_2, \quad [w_2]_{S_W} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Step 3

for W

so

$$P_{S_W \leftarrow C_W} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$