

Answer any seven questions, show your work

- ① Compute the mean, Gaussian, principal curvatures of the surface  $z = x^2 - y^2$  at  $(0, 0, 0)$ .
- ② State and prove the Meusnier Theorem.
- ③ Let  $S$  be a connected regular surface in  $\mathbb{R}^3$  that is umbilical at every point. Prove that the Gaussian curvature of  $S$  is a constant.
- ④ Let  $\alpha(s)$  be a unit speed curve in  $\mathbb{R}^3$ . Prove that its torsion  $\equiv 0$  iff it is planar.
- ⑤
  - (a) State the Isoperimetric Inequality
  - (b) Is there a simple closed curve  $C$  in the plane with length equal to 5 ft bounding an area 2 square feet? Give reasons.
  - (c) In (b) what is the maximum area that  $C$  could bound? What is this curve?
- ⑥ Let  $S$  be a compact regular surface in  $\mathbb{R}^3$ . Prove that its mean curvature cannot vanish everywhere
- ⑦
  - (a) State the fundamental theorem for the local theory of curves
  - (b) Prove the uniqueness part (i.e. rigidity theorem) of (a).