Homework 13, due 5/26, 10pm

This is the exercise 8.8 in textbook. Here I give you more hints.

1. [5pts] Let $k \neq 0$, show that the function $G_k(x,\xi) = e^{-k|x-\xi|}/2k$ is a fundamental solution of the equation

$$-u'' + k^2 u = 0, \quad -\infty < x < \infty.$$

By definition of fundamental solution, this is to show that $G_k(x,\xi)$ satisfies

$$-G_k'' + k^2 G_k = \delta(x-\xi), \quad -\infty < x < \infty.$$

Hint, use some Green's identities (here it is integration by parts), see also lecture 14 for the derivation of $\Delta\Gamma = -\delta$.