HW: 13

If $x < \xi$, $G = e^{-k(\xi - x)} / 2k$

$$G'' = \frac{1}{2k} e^{-k(\xi - x)} \cdot k^2$$

$$-G'' + k^2 G = -\frac{k}{2} e^{-k(\xi - x)} + k^2 \cdot \frac{1}{2k} e^{-k(\xi - x)} = 0$$

If $x > \xi$, $\cdots$ $-G'' + k^2 G = 0.$

$$\int_{-\infty}^{\infty} u(x)\left(-G'' + k^2 G\right) dx$$

$$= \int_{\xi - \varepsilon}^{\xi + \varepsilon} u(x)\left[-G'' + k^2 G\right] dx$$

$$= \underbrace{-uG'\Big|_{\xi - \varepsilon}^{\xi + \varepsilon}}_{(i)} + \underbrace{\int_{\xi - \varepsilon}^{\xi + \varepsilon} u'G' + k^2 G u \, dx}_{(ii)}$$

(i) $$G_k' = \begin{cases} \frac{1}{2} e^{-k(\xi - x)} & x < \xi \\ -\frac{1}{2} e^{-k(x-\xi)} & x > \xi \end{cases}$$

$$-u\, G' \Big|_{\xi-\varepsilon}^{\xi+\varepsilon}$$

$$= -u(\xi+\varepsilon)\left(-\frac{1}{2}\right) e^{-k\varepsilon}$$

$$+ u(\xi-\varepsilon) \cdot \frac{1}{2} e^{-k\varepsilon}$$

$$\longrightarrow u(\xi) \qquad \text{as } \varepsilon \to 0.$$

(ii) $|u'| < C$, $|G'| < C$ $|u| < C$, $|G| < C$

$$\Longrightarrow \int_{\xi-\varepsilon}^{\xi+\varepsilon} u'G' + k^2 u G \, dx \to 0, \text{ as } \varepsilon \to 0.$$

So $$\int_{-\infty}^{\infty} u \cdot (-G'' + kG)\, dx$$

$$= u(\xi)$$

$$\Rightarrow -G'' + kG = \delta(x - \xi)$$