

HW: 13

$$\text{If } x < \xi, \quad G = e^{-k(\xi-x)} / 2k$$

$$G'' = \frac{1}{2k} e^{-k(\xi-x)} \cdot k^2$$

$$-G'' + k^2 G = -\frac{k}{2} e^{-k(\xi-x)} + k^2 \cdot \frac{1}{2k} e^{-k(\xi-x)} = 0$$

$$\text{If } x > \xi, \quad \dots \quad -G'' + k^2 G = 0.$$

$$\begin{aligned} & \int_{-\infty}^{\infty} u(x) (-G'' + k^2 G) dx \\ &= \int_{\xi-\varepsilon}^{\xi+\varepsilon} u(x) [-G'' + k^2 G] dx \\ &= \underbrace{-uG' \Big|_{\xi-\varepsilon}^{\xi+\varepsilon}}_{(i)} + \underbrace{\int_{\xi-\varepsilon}^{\xi+\varepsilon} u'G' + k^2 Gu dx}_{(ii)} \end{aligned}$$

$$(i) \quad G_k' = \begin{cases} \frac{1}{2} e^{-k(\xi-x)} & x < \xi \\ -\frac{1}{2} e^{-k(x-\xi)} & x > \xi \end{cases}$$

$$\begin{aligned} & -u G' \Big|_{\xi-\varepsilon}^{\xi+\varepsilon} \\ &= -u(\xi+\varepsilon) \left(-\frac{1}{2}\right) e^{-k\varepsilon} \\ & \quad + u(\xi-\varepsilon) \cdot \frac{1}{2} e^{-k\varepsilon} \\ & \rightarrow u(\xi) \quad \text{as } \varepsilon \rightarrow 0. \end{aligned}$$

$$(ii) \quad |u'| < C, \quad |G'| < C \quad |u| < C, \quad |G| < C \\ \Rightarrow \int_{\xi-\varepsilon}^{\xi+\varepsilon} u G' + k^2 u G \, dx \rightarrow 0, \quad \text{as } \varepsilon \rightarrow 0.$$

$$\begin{aligned} \text{So } \int_{-\infty}^{\infty} u \cdot (-G'' + kG) \, dx \\ = u(\xi) \end{aligned}$$

$$\Rightarrow -G'' + kG = \delta(x-\xi)$$