HW.

$$\begin{cases} y u_x - x u_y + u_z = u^2 \\ u(x,y,0) = \frac{1}{y} \end{cases}$$

(1)

$$\begin{cases} x_t = y \\ y_t = -x \\ z_t = 1 \\ u_t = u^2 \end{cases} \qquad \begin{cases} x(0,s_1,s_2) = s_1 \\ y(0,s_1,s_2) = s_2 \\ z(0,s_1,s_2) = 0 \\ u(0,s_1,s_2) = \frac{1}{s_2} \end{cases}$$

$$z = t$$

$$\frac{du}{u^2} = dt \Rightarrow -\frac{1}{u} = t + C$$

$$u = \frac{-1}{t+C}$$

at $t=0 \Rightarrow \frac{-1}{C} = \frac{1}{s_2} \Rightarrow C = -s_2$

$$u = \frac{-1}{t - s_2}$$

$$x_{tt} = -x$$

$$x = C_1 \cos t + C_2 \sin t$$

$$y = -C_1 \sin t + C_2 \cos t$$

$$C_1 = S_1, \quad C_2 = S_2$$

$$x = S_1 \cos t + S_2 \sin t$$

$$y = -S_1 \sin t + S_2 \cos t.$$

$$\Rightarrow \quad x \sin t + y \cos t = S_2$$

$$\Rightarrow \quad S_2 = x \sin(z) + y \cos(z)$$

$$\Rightarrow \quad u = \frac{-11}{z - x\sin(z) - y\cos(z)}$$

(2)

$$J = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ S_2 & -S_1 & 1 \end{vmatrix} = 1 \neq 0.$$

Yes. There is a unique solution.