HW6. [key steps]

1: $\dfrac{X_{xx}}{X} = \dfrac{T_t}{kT} = -\lambda$

$$X_{xx} + \lambda X = 0. \quad (1)$$

$$T_t + \lambda k T = 0 \quad (2)$$

$$X(0) = X(2\pi), \quad X_x(0) = X_x(2\pi). \quad (3).$$

① $\lambda < 0$ $\quad X = C_1 e^{\sqrt{-\lambda}x} + C_2 e^{-\sqrt{-\lambda}x}.$

$$C_1 = C_2 = 0$$

② $\lambda = 0.$ $\quad X = C_1 x + C_2$

$$C_1 = 0.$$

$$X_0 = 1.$$

$$T_0 = \frac{A_0}{2} \Rightarrow u_0 = \frac{A_0}{2}$$

,

③ $\lambda > 0$. $X = C_1 \cos(\sqrt{\lambda} x) + C_2 \sin(\sqrt{\lambda} x)$

$X_x = -C_1 \sqrt{\lambda} \sin(\sqrt{\lambda} x) + C_2 \sqrt{\lambda} \cos(\sqrt{\lambda} x)$.

$$\begin{cases} C_1 = C_1 \cos(2\pi\sqrt{\lambda}) + C_2 \sin(2\pi\sqrt{\lambda}), \\ C_2 = -C_1 \sin(2\pi\sqrt{\lambda}) + C_2 \cos(2\pi\sqrt{\lambda}). \end{cases}$$

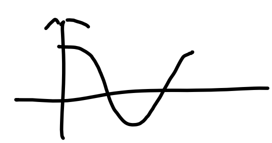
$\alpha = 2\pi\sqrt{\lambda}$ $$\begin{vmatrix} \cos(2\pi\sqrt{\lambda}) - 1, & \sin(2\pi\sqrt{\lambda}) \\ -\sin(2\pi\sqrt{\lambda}), & \cos(2\pi\sqrt{\lambda}) - 1 \end{vmatrix}$$

$= (\cos\alpha - 1)^2 + \sin^2\alpha$

$= \cos^2\alpha - 2\cos\alpha + 1 + \sin^2\alpha$

$= 2(1 - \cos\alpha) = 0$.

$\cos\alpha = 1$.



$2\pi\sqrt{\lambda} = 2\pi n$.

$\lambda = n^2$.

C_1 & C_2 can be arbitrary.

$X_n = C_{1n} \cos(nx) + C_{2n} \sin(nx)$.

$$T_t + n^2 k T = 0$$

$$T_n = D_n e^{-n^2 k t}$$

$$u = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(nx) + B_n \sin(nx)\right] \underline{\underline{e^{-n^2 k t}}}$$

$$u(x,0) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(nx) + B_n \sin(nx)\right].$$

$$= f(x)$$

is just the Fourier series of $f(x)$.

$$A_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx)\, dx, \quad n = 0, 1, 2, 3, \cdots$$

$$B_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(nx)\, dx, \quad n = 1, 2, 3, \cdots$$

[part (b) not needed.]