Review.

① For $u_t = k u_{xx}$ (1)

$u_{tt} = c^2 u_{xx}$ (2)

$[u_{xx} + u_{yy} = 0]$

If $u(0,t) = u(L,t) = 0$

$[\text{or } u(0,y) = u(L,y) = 0]$

$\rightarrow X_n = \sin\left(\frac{n\pi}{L}x\right)$

$n = 1, 2, \cdots$

If $u_x(0,t) = u_x(L,t) = 0$

$[\text{or } u_x(0,y) = u_x(L,y) = 0]$

$\rightarrow X_n = \cos\left(\frac{n\pi}{L}x\right)$

$n = 0, 1, 2, \cdots$

Similarly for periodic BC $\longrightarrow X_n = \cdots$

RK1: · Do not need to derive X_n again.

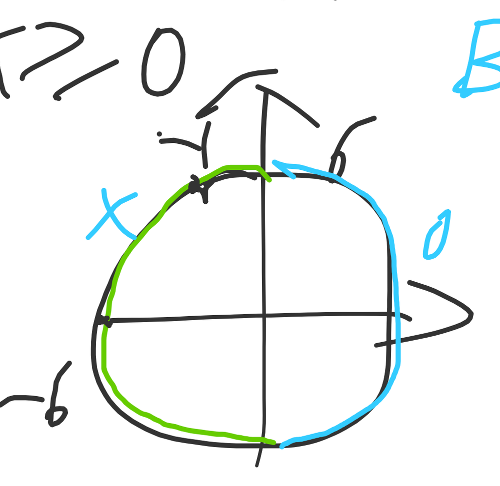
RK2: use general solution in lectures directly

② Ex 7.22

$\Delta u = 0$ in D PDE

$u = \begin{cases} x, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases}$ on ∂D BC

$D: x^2 + y^2 < 36$



(a) Prove $u(x,y) \leq \min\{x, 0\}$

(i) By (weak) MP

$\max_D u \leq \max_{\partial D}(u) = 0$

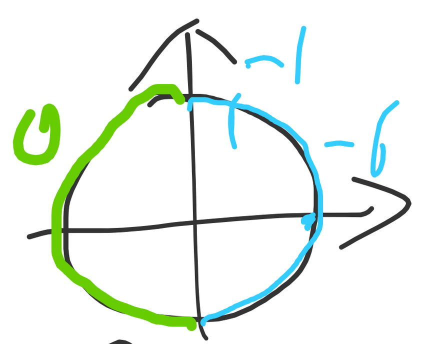
$\Rightarrow u \leq 0$ in D (1)

(ii) Let $v = u - x$

$\Delta v = \Delta u - \Delta x$

$= 0 - 0 = 0$ (PDE)

$v = \begin{cases} 0, & \text{if } x < 0 \\ -x, & \text{if } x \geq 0 \end{cases}$ (BC) on ∂D.



By MP on system of v.

$\max_D v \leq \max_{\partial D}(v) = 0$

$\Leftrightarrow \max_D (u - x) \leq 0$

$\Rightarrow u - x \leq 0$ in D

$\Rightarrow u \leq x$ in D (2)

Combining (1) & (2)

$u \leq \min\{0, x\}$ in D.