MIDTERM
MATH 146C Section 10
Spring 2020

Please write the pledge below in your own handwriting, fill in the required information, and sign. "On my honor, I (your name) have neither given nor received any unauthorized aid on this examination. I have never used unauthorized tools or look up unauthorized materials. I finish my exam within 80 minutes"

- Pledge:

- First Name:
- Last Name:
- Student ID:
- Sign:

Instructions:

- Write above pledge and sign, and without this pledge you will get a 0 for final exam.
- **Show ALL your work to receive credit!** Unless otherwise specified, an answer without explanation might receive no credit.
- This is open book. You can use lecture notes, your own notes, or books. But you can not use calculators, and can not search answers online or receive help from others.
- You have 80 minutes to complete this exam and 20 minutes to submit to crowdmark.

Question:	1	2	3	4	5	Total
Points:	7	7	6	5	5	30

Question 1 (7 points) For the following partial differential equation (PDE) for $u(x, y)$

$$u_{xx} + u_{yy} = x^2 + y^2$$

(a). (4 points) What is the order of the PDE? Is it a linear PDE or nonlinear PDE?

(b). (3 points) Suppose $u(x, y)$ has a solution in the form $u(x, y) = h(r)$, where $r = \sqrt{x^2 + y^2}$, find the ordinary differential equation (ODE) satisfied for $h(r)$. Be sure to show your derivation.

Question 2 (7 points) Solve the first order PDE by method of characteristics

$$u_x + 2u_y = 3, \quad u(x,0) = x$$

Question 3 (6 points) For the first order PDE

$$u_x + 3u_y = u^3, \quad u(x, 3x) = 1$$

(a). (2 points) find the charateristics L_p, i.e., the projection of characteristic curves on x-y plane.

(b). (4 points) discuss the existence and uniqueness of the solution.

Question 4 (5 points) Solve the problem

$$\begin{aligned} u_t - u_{xx} &= \cos(t) && 0 < x < 1, t > 0, \\ u_x(0,t) &= u_x(1,t) = 0, && t \geq 0, \\ u(x,0) &= 1 + \cos(\pi x), && 0 \leq x \leq 1, \end{aligned}$$

[Hint: use the eigenfunctions $X_n = \cos(n\pi x)$, $(n = 0, 1, 2, ...)$ in your expansion]

Question 5 (5 points) Solve the problem

$$
\begin{aligned}
&u_{tt} = u_{xx}, && 0 < x < 1, t > 0,\\
&u_x(0,t) = u_x(1,t) = 0, && t \geq 0,\\
&u(x,0) = 1 + \cos(\pi x), && 0 \leq x \leq 1,\\
&u_t(x,0) = \cos(\pi x)\cos(2\pi x), && 0 \leq x \leq 1
\end{aligned}
$$

[Hint: you may directly use the derived solution in lecture]