MATH 147 Discussion
Quiz 3
March 5, 2021

Directions: Write your solution to Question 1 directly on this page[1] and your solutions to Questions 2 and 3 on a separate sheet of paper. Once you are finished with the quiz, take pictures of your solutions to each question **separately**, and submit your quiz solutions on Crowdmark, separated by question (Q1, Q2, Q3). Please note that you must submit your quiz by 1:10 p.m. deadline, unless I give a time extension to everyone.

(5pts) 1. Find in the following word search all ten words of key terms from Chapters 5 and 6 of the Stein and Shakarchi textbook. The words are placed horizontally, vertically, or diagonally, as well as forward or backward. Each word is worth one-half of a point.

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H F D L M A A K M N O I S R E V N I R E I R U O F H Y R L D T J H C S
V F M I R O R W W R J Y O X W X L D Q V V Y K B P B G A N T S Z M P U
B N U A Q O D T R R O C G Z N V H N P Q Z M C E W Q F P N X K N I L B
P U P N T Q M E P V N F D D S G X E B N W E C K M X R I Y K I O G T E
C H N P F H N S R F X X S G L B U V A B A A C O F K U D U E W I V D Q
A F B K W U S E D A Z O Y N V G P A F T P G S B W S J L X R N T R L L
E B D M C W A O G V T S X A A E N O H S K F L F P J D Y H E E U P C H
H P R T K J V A J I H E B O H R B B Z Y M E T D R H M D P K Y L S N D
Z V K A P V U J K N B D D A K U T R J F S G R D Z F Z E X T W O F Z J
H O U P D S L D U M Z O L E N L A R H A U B L N G T I C S R V V M J V
F S M J S I O U Q N S U X V C W T M E B G S P Q E E Z R E R H N E P W
B N D I O W A P I S E A F A H R L A G I P O L V S L U E V Y Z O Z A H
I J A G K F L L K E Y Z K C K A E K V U R M G Q R A A A M M E C U D W
C N Z N I B J B Q Y X D S I W L E A J H O U H N L N D S J J G O G L F
I Y S B B L C M D F A H Q E A N W O S D Y I O K G A X I P K Q W A H Y
Z M L I F N M Y H B Z Y R L V J M G Z E R G E F P T C N P B U P H D Z
N S T X D F T R A N S L A T I O N I N V A R I A N C E G U Y O X F U L
E B B A J F X R A C Q F L H C D V G F G Q L E P J T P J M Y H S V S Q
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Instead of a word bank, the definitions corresponding to the ten words are provided below.

- This process describes going from f to $\hat{f}$.
- This defines the property $\int_{-\infty}^{\infty} f(x-h)\,dx = \int_{-\infty}^{\infty} f(x)\,dx$ for any $h \in \mathbb{R}$.
- What is the name of the set $\mathcal{S}(\mathbb{R})$?
- What is the name of $f \in \mathcal{S}(\mathbb{R})$ defined by $f(x) = e^{-x^2}$?
- This kind of f defined on $\mathbb{R}$ satisfies $|f(x)| \leq \frac{A}{1+x^2}$ for some $A > 0$.
- This kind of f satisfies $\sup_{x \in \mathbb{R}} |x|^k |f^{(\ell)}| < \infty$ for all integers $k, l \geq 0$.
- If $f, g \in \mathcal{S}(\mathbb{R})$, then $(f * g)(x) = \int_{-\infty}^{\infty} f(x-t)g(t)\,dt$.
- This process describes going from $\hat{f}$ to f.
- The solution of the heat equation is $u(x,t) = (f * \mathcal{H}_t)(x)$, where $\mathcal{H}_t$ is called this.
- A function $f \in \mathcal{S}(\mathbb{R}^d)$ for any integer $d \geq 1$ is called this if it depends only on $|x|$.

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[1]If for any reason you must write your solution to this question on a separate sheet of paper, please write the (i, j)-entries corresponding to the start and end of a word. For instance, the word MATH starts at row 2, column 3 and ends at row 5, column 6; you would write your answer to that word on your separate sheet as `(2,3)->(5,6)`.

(8pts) 2. Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \chi_{[0,1]}(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(4pts) (a) Compute the expression of the Fourier transform $\hat{f}$ over $\mathbb{R}$.

Note: Your final answer should be expressed in terms of a sine function.

(4pts) (b) Show that $\hat{f}$ is continuous and bounded on $\mathbb{R}$.

Note: You may use without proof any known rules of continuity and the basic properties of sine and cosine.

(2pts) 3. We had such an exciting time in Fourier analysis this quarter. What was *your* favorite part of the course?[2]

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[2]I like that too.