

Notes for Week 1 discussion on October 1

Definition. An outcome is a result of an experiment.

Example. A coin toss has 2 outcomes, which are heads (H) and tails (T).

Example. Rolling a six-sided die has 6 outcomes, which can be represented by the numbers 1, 2, 3, 4, 5, 6.

Example. Rolling two six-sided dice has 36 outcomes, which can be represented by an array of coordinate pairs

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Definition. A sample space is the set of all possible outcomes.

Example. Here are the sample spaces of the three experiments we discussed above:

- Coin toss: $C = \{H, T\}$,
- Rolling a six-sided die: $C = \{1, 2, 3, 4, 5, 6\}$,
- Rolling two six-sided dice: $C = \{(i, j) : i, j \in \{1, 2, 3, 4, 5, 6\}\}$. Note that, for purposes of brevity, we have just used set-builder notation for this C to describe all the 36 coordinate pairs $(1, 1), \dots, (1, 6), (2, 1), \dots, (2, 6), \dots, (6, 1), \dots, (6, 6)$ as written explicitly above.

In general, we let:

- C be the sample space,
- c be an outcome; that is, $c \in C$,
- $C \subset C$ be a set containing some or all of the outcomes c .

Definition. If we have $c \in C$, then we say that an event C has occurred.

The next three examples will look at our previous experiments once again.

Example. Let $C = \{H, T\}$ be the sample space. Let $C_1 = \{H\}$ and $C_2 = \{T\}$. If you flip a coin and get heads, then event C_1 has occurred and event C_2 has not occurred. Likewise, if you flip a coin and get tails, then event C_2 has occurred and event C_1 has not occurred.

Example. Let $C = \{1, 2, 3, 4, 5, 6\}$ be the sample space, and let $C = \{1, 2\}$. If you roll a 1 or 2, then event C has occurred. However, if you roll any one of 3, 4, 5, 6, then event C has not occurred.

Example. Let $C = \{(i, j) : i, j \in \{1, 2, 3, 4, 5, 6\}\}$ be the sample space, and let $C = \{(i, j) : i + j = 8\}$. If you roll a 6 and a 2, then event C has occurred because we have $6 + 2 = 8$. If you roll a 5 and a 3, then event C has occurred because we have $5 + 3 = 8$. If you roll a 5 and a 4, then event C has not occurred because we have $5 + 4 = 9 \neq 8$. We can explicitly write $C = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$.