HOMEWORK ASSIGNMENT ONE

MATH 150A, WINTER 2020

1. Consider the following quesitons concerning bounded set.
   (1) State the definition of a bounded set $S \subset \mathbb{R}$.
   (2) Show that $S = \{1, \ldots, 10\} \cup (-1, 1)$ is bounded.
   (3) Show that the set of natural numbers $\mathbb{N}$ is an unbounded subset of $\mathbb{R}$.
   (4) Show that the union of two bounded set is bounded.

2. We have triangle inequality: $|x + y| \leq |x| + |y|$ for all $x, y \in \mathbb{R}$.
   (1) When the inequality becomes equality, i.e. $|x + y| = |x| + |y|$?
   (2) When it becomes a strict inequality, i.e. $|x + y| < |x| + |y|$?
   (3) Show that $|x - y| \leq |x| + |y|$ for all $x, y \in \mathbb{R}$ via the triangle inequality.
   (4) Show that $\big||x| - |y|\big| \leq |x - y|$ for all $x, y \in \mathbb{R}$ via the triangle inequality.

3. Consider the following questions concerning the defition of convergent sequences.
   (1) State the definition of convergent sequences.
   (2) Use the definition to show that if $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} |a_n| = |L|$.
   (3) Is it true that if $\lim_{n\to\infty} |a_n| = |L|$, then $\lim_{n\to\infty} a_n = L$? Why?

4. Try to mimick the class examples to show convergence of the following sequences by definition.
   (1) $\lim_{n\to\infty} \frac{1}{n^2} = 0$.
   (2) $\lim_{n\to\infty} \frac{1}{2^n} = 0$
   (3) Let $0 < a < 1$. Show that $\lim_{n\to\infty} n^2 a^n = 0$.