HOMEWORK ASSIGNMENT THREE

MATH 150A, WINTER 2020

1. Try to write down the proof of the sum and product limit laws without consulting any resources.

2. Consider the convergenct of the following sequences. If convergent, compute their limits and justify your steps. If divergent, explain your reasoning. You may directly use the class examples or the examples from previous homework assignments.

$$\left\{\frac{n3^n + n^2 2^n + 5n}{5^n + n^3}\right\},\ \left\{\sqrt{n+2} - \sqrt{n}\right\},\ \left\{\frac{n^2+1}{n}\right\}.$$

3. Use the class example $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ and the formula

$$1 + \frac{3}{n} = \left(1 + \frac{1}{n}\right)\left(1 + \frac{1}{n+1}\right)\left(1 + \frac{1}{n+2}\right)$$

to show that

$$\lim_{n\to\infty}\left(1 + \frac{3}{n}\right)^n = e^3.$$

4. Define a sequence by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$ for all $n \geq 1$. Prove by induction that $\{a_n\}$ is monotone increasing and bounded above by 2 so that it's convergent. Compute the limit.