HOMEWORK ASSIGNMENT FOUR

MATH 150A, WINTER 2020

1. Write down the definitions of limit point, limit set, closure, and closed set without consulting any resources. Think about the relations of these notions. **Repeat the process again and again until you are able to write down exactly the same statements as we did in class or as written in the textbook.**

2. Apply Theorem 1.14 (in my lecture notes) to find the closure of the set

$$S = \left\{ \frac{1}{n} : \ n \in \mathbb{Z}_+ \right\}.$$

Remark: note here you need to find all the limit points of S.

3. Try the following questions: (i) show that a closed interval is a closed set; (ii) show that the set of integers $\mathbb{Z}$ is a closed set; (iii) based on your proof of part (i), find an unbounded set that has no limit point.

I have done part (i) in class, I put it here to make sure that you understand my proof. Make sure that eventually you can prove it without looking at my proof.

4. Try to mimick the proof of Theorem 1.15 to do the following problem:

Let $\{I_n\}_{n\in\mathbb{Z}_+}$ be a sequence of closed intervals, and denote by λ_n the length of I_n. Assume that it's decreasing in the sense that

$$I_1 \supset I_2 \supset \cdots I_n \supset I_{n+1} \supset \cdots .$$

Show that (i) $\lim \lambda_n$ exists; (ii) if $\lim \lambda_n > 0$, then the intersection of those intervals, $\bigcap_{n\in\mathbb{Z}_+} I_n$, is a closed interval with length $\lim \lambda_n$.