

HOMEWORK ASSIGNMENT FIVE

MATH 150A, WINTER 2020

1. Let $f : I \rightarrow \mathbb{R}$ be a function defined on an interval $I \subset \mathbb{R}$. Write down the definitions of $\lim_{x \rightarrow \xi} f(x) = A$, $\lim_{x \rightarrow \xi+} f(x) = A$, $\lim_{x \rightarrow \xi-} f(x) = A$, and the definition of f is continuous at ξ . What does it mean by saying f is continuous on I ? **Repeat the process again and again until you can easily write down exactly the same statements as we did in class or as written in the textbook.**

2. Show by definition that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x|$, is continuous on $\mathbb{R}$.

Hint: you may need to use the triangle inequality.

3. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ to be

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0. \end{cases}$$

(1) Show by definition that f is continuous at every $x \neq 0$.

(2) Compute by definition $\lim_{x \rightarrow 0+} f(x)$ and $\lim_{x \rightarrow 0-} f(x)$.

(3) Show that f is discontinuous at $x = 0$.

Hint: you may need to follow the proof of continuity of constant functions

4. Show by definition that $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \cos(x)$, is continuous on $\mathbb{R}$.

Hint: you may need to mimic the proof of continuity of $f(x) = \sin(x)$ on $\mathbb{R}$.