HOMEWORK ASSIGNMENT SIX

MATH 150A, WINTER 2020

1. Show by defintion that $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \cos(x)$, is continuous on $\mathbb{R}$.

   Hint: You may try to mimick the proof of conitinuity of $f(x) = \sin(x)$ on $\mathbb{R}$. You may need to use the formula
   $$\cos a - \cos b = -2 \sin \frac{a+b}{2} \cdot \sin \frac{a-b}{2}$$
   .

2. Show by definition that the function $f : [0, \infty) \to \mathbb{R}$ where $f(x) = \sqrt{x}$ is continuous on $\mathbb{R}$.

   Hint: you have to consider two differen cases: (i) continuity at $\xi > 0$; (ii) continuity at $\xi = 0$. Note in the second case, one can only approach $\xi = 0$ from right, i.e. case (ii) is basically $\lim\limits_{x \to 0+} f(x) = f(0)$.

3. Consider the function $f(x) : (0, \infty) \to \mathbb{R}$ where $f(x) = \frac{1}{x}$.
   (1) Show by definition that f is continuous at every $\xi > 0$.
   (2) Show that f does not have a right limit at $\xi = 0$. Here you may use Theorem 2.2 and its corollaries.

4. Prove Theorem 2.4 via Theorem 2.2.