

HOMWORK ASSIGNMENT SEVEN

MATH 150A, WINTER 2020

1. Compute the following limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{x}$$

Hint: Note this function is not defined at $x = 0$. But you may try to convert it into a function that is continuous on $\mathbb{R}$. Then use that function to compute the limit.

2. Prove that if $f : I \rightarrow \mathbb{R}$ is continuous on I , then $|f| : I \rightarrow \mathbb{R}$ where

$$|f|(x) := |f(x)| \quad \forall x \in I$$

is continuous on I . Then find a function $f(x)$ so that $|f(x)|$ is continuous at some ξ while $f(x)$ is discontinuous at ξ .

3. Show that the function $f(x) = \cos(\sqrt{x^2 + (\frac{\pi}{2})^2})$ is continuous on $\mathbb{R}$. Then compute the following limit

$$\lim_{n \rightarrow \infty} \cos \left(\sqrt{\frac{1}{n^2} + \left(\frac{\pi}{2}\right)^2} \right)$$

4. Construct following three types of functions.

- (1) $f : I \rightarrow \mathbb{R}$ is continuous and I is bounded. But f is unbounded.
- (2) $f : I \rightarrow \mathbb{R}$ where I is closed and bounded. But f is unbounded.
- (3) $f : I \rightarrow \mathbb{R}$ is continuous and I is bounded. f has a minimum point but doesn't have a maximum point.