

**MATH 150A-QUIZ 6, WINTER 2020**

Name: \_\_\_\_\_

1. (5 pts) Let  $f, g : I \rightarrow \mathbb{R}$  both be continuous at  $\xi$ . Show that the product  $fg$  is continuous at  $\xi$ .

*Here you may either prove it by definition or prove it via the following theorem that we proved in class (Theorem 2.2):  $h : I \rightarrow \mathbb{R}$  is continuous at  $\xi \in I$  if and only if the following statement holds true: for any sequence  $\{x_n\}$  in  $I$  with  $\lim_{n \rightarrow \infty} x_n = \xi$ , it holds that  $\lim_{n \rightarrow \infty} h(x_n) = h(\xi)$ .*

2. Consider a function  $f(x) = \cos(x)$ .

(1) (1 pts ) State the definition that  $f$  is continuous at a  $\xi \in \mathbb{R}$ . Then state the definition that  $f$  is continuous on  $\mathbb{R}$ .

(2) (4 pts) Show by definition that  $f$  is continuous on  $\mathbb{R}$ .

*You may need to use the facts:* (1)  $|\sin(x)| \leq |x|, \forall x \in \mathbb{R}$ ; (2)  $|\sin(x)| \leq 1, \forall x \in \mathbb{R}$ ;

$$(3) \cos a - \cos b = -2 \sin \frac{a+b}{2} \cdot \sin \frac{a-b}{2} \quad \forall a, b \in \mathbb{R}.$$