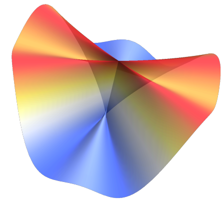
UC RIVERSIDE
Department of Mathematics

MATH 151B - Advanced Calculus



HIGHER STAKES HOMEWORK 1-2

Due date: Wednesday, January 27 at 11:59pm

Instructions:

* Work individually in the problems. You can ask questions to Estela, Chulan, or Ryan
* You can use any book, article or web-based mathematical material or computational software
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The homework needs to be typeset in "LaTeX" and uploaded through Gradescope in the iLearn Lecture page
* If a problem is similar to a problem in Hw #1 or Hw #2, you need to adapt the proof for this problem, not just refer or reproduce all the solution of the problem in the homework.

- **Problem 1:** Given the sequence of functions

$$f_n : [0,1] \to \mathbb{R}$$
$$f_n(x) = \frac{x}{1+nx}.$$

(a) Find the pointwise limit of the sequence and prove this limit.

(b) Is the convergence uniform? Why?

- **Problem 2:** Let the functions of functions $\{h_n\}$ defined by

$$h_n : [0,1) \to \mathbb{R}$$
$$h_n(x) = \frac{x^n}{1+x^{2n}}.$$

Does $\{h_n\}$ converge uniformly on $[0,1)$? Why?

- **Problem 3:** Find a sequence of continuous functions, $f_n : [0,1] \to \mathbb{R}$ such that

$$\lim_{x\to 1}\left(\lim_{n\to\infty} f_n(x)\right) \text{ and } \lim_{n\to\infty}\left(\lim_{x\to 1} f_n(x)\right) \text{ exist and}$$

$$\lim_{x\to 1}\left(\lim_{n\to\infty} f_n(x)\right) \neq \lim_{n\to\infty}\left(\lim_{x\to 1} f_n(x)\right).$$

- **Problem 4:** Let $f_n : [0,\infty) \to \mathbb{R}$ be sequence of functions that are Riemann integrable on any interval of the form $[0,x]$, $x > 0$. Assume that f_n converges uniformly to a function f on $[0,\infty)$. Define the "average" sequence of functions:

$$g_n : (0,\infty) \to \mathbb{R}$$
$$g_n(x) = \frac{1}{x}\int_0^x f_n(t)dt$$

and the function

$$g : (0,\infty) \to \mathbb{R}$$
$$g(x) = \frac{1}{x}\int_0^x f(t)dt$$

Prove that g_n converges uniformly to g on $(0,\infty)$.