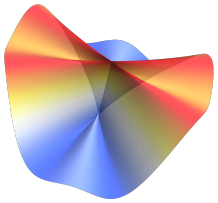
UC RIVERSIDE
Department of Mathematics

MATH 151B - Advanced Calculus



HIGHER STAKES HOMEWORK 1-2

Due date: Tuesday, February 16 at 11:59pm

Instructions:

* Work individually in the problems. You can ask questions to Estela, Chulan, or Ryan
* You can use any book, article or web-based mathematical material or computational software
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The homework needs to be typeset in "LaTeX" and uploaded through Gradescope in the iLearn Lecture page
* If a problem is similar to a problem in Hw #3 or Hw #4, you need to adapt the proof for this problem, not just refer or reproduce all the solution of the problem in the homework.

- **Problem 1:** Let $C([a,b]) = \{f : [a,b] \to \mathbb{R}, f \text{ continuous } \}$. Show that

$$d(f,g) = \int_a^b \frac{|f(x) - g(x)|}{1 + |f(x) - g(x)|} dx$$

is a metric on $C([a,b])$.

- **Problem 2:** Consider the sequences $\{f_n\}$ and $\{f_n'\}$, where $f_n(x) = \frac{1}{n}\exp\left(-n^2x^2\right)$ on the interval [-1,1] . Show whether $\{f_n\}$ converges pointwise, uniformily, or if it diverges at some point. Do the same with $\{f_n'\}$. Justify your answers.

- **Problem 3:** For what values of $a > 0$ is the power series $\sum_{n=1}^{\infty} e^{-2n} x^n$ uniformly convergent in the interval $[0, a]$?

- **Problem 4:** Let X be a metric space, $E \subset X$ be closed, and let $\{x_n\}$ be a sequence in X converging to $p \in X$. Suppose $x_n \in E$ for infinitely many $n \in \mathbb{N}$. Show $p \in E$.