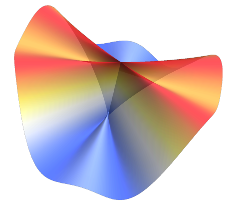
UC RIVERSIDE
Department of Mathematics

MATH 151B - Advanced Calculus



HIGHER STAKES HOMEWORK 3

Due date: Wednesday, March 10 at 11:59pm

Instructions:

∗ Work individually in the problems. You can ask questions to Estela, Chulan, or Ryan

∗ You can use any book, article or web-based mathematical material or computational software

∗ Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed

∗ The homework needs to be typeset in "LaTeX" and uploaded through Gradescope in the iLearn Lecture page

∗ If a problem is similar to a problem in Hw #5 or Hw #6, you need to adapt the proof for this problem, not just refer or reproduce all the solution of the problem in the homework.

- **Problem 1:** Let $f : \mathbf{R}^2 \to \mathbf{R}$ be given by

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Show that the first-order partial derivatives of f, f_x and f_y, exist at (0,0) . Show, however, that f is not differentiable at $(0,0)$.

- **Problem 2:** Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} x^2 + y^2 & \text{if } x \text{ and } y \text{ are rational} \\ 0 & \text{otherwise.} \end{cases}$$

Find all the points $(x,y) \in \mathbb{R}^2$ (if any) where f is differentiable. Justify your answer.

- **Problem 3:** Let $f:(-1,1)\to\mathbb{R}$ be differentiable, so that $f(0)=0, f'(0)\neq 0$. Show that the function $F(x,y)=(u(x,y),v(x,y))$

$$\left\{\begin{array}{rc} u(x,y)= & f(x) \\ v(x,y)= & -y+xf(x). \end{array}\right.$$

is locally invertible near (0,0) and its inverse has the form

$$\left\{\begin{array}{rc} x(u,v)= & g(u) \\ y(u,v)= & -v+ug(u). \end{array}\right.$$

- **Problem 4:** Define $f : \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x,y,z) = x^2 y + 2e^x + z.$$

Prove there exists a differentiable function g defined in some neighborhood $\mathcal{B} \subset \mathbb{R}^2$ of (1,-2) such that $g(1,-2) = 0$ and

$$f(g(y,z),y,z) = 0$$

for all $(y,z) \in \mathcal{B}$. Furthermore, evaluate $\frac{\partial g}{\partial y}(1,-2)$ and $\frac{\partial g}{\partial z}(1,-2)$.