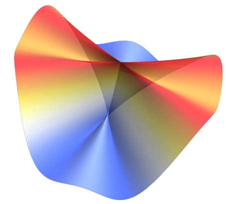
UC RIVERSIDE
Department of Mathematics

MATH 151B - Advanced Calculus



Homework 1

Due date: Monday, January 11 at 11:59pm

Instructions:

* You are encourage to work in group but you have to write your individual solution
* You can use any book, article or web-based mathematical material or computational software
* You can ask Chulan, Ryan, or Estela questions
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The homework needs to be typeset in "LaTeX" and uploaded through Gradescope in the iLearn Lecture page

Problems

1. (Exercise 6.1.2)

   a) Find the pointwise limit $\dfrac{e^{x/n}}{n}$ for $x \in \mathbb{R}$.

   b) Is the limit uniform on $\mathbb{R}$?

   c) Is the limit uniform on $[0, 1]$?

2. (Exercise 6.1.6) Find an example of a sequence of functions $\{f_n\}$ and $\{g_n\}$ that converge uniformly to some f and g on some set A, but such that $\{f_n g_n\}$ (the multiple) does not converge uniformly to fg on A.

   Hint: Let $A := \mathbb{R}$, let $f(x) := g(x) := x$. You can even pick $f_n = g_n$.

3. (Exercise 6.1.7) Suppose there exists a sequence of functions $\{g_n\}$ uniformly converging to 0 on A. Now suppose we have a sequence of functions $\{f_n\}$ and a function f on A such that

$$|f_n(x) - f(x)| \leq g_n(x)$$

for all $x \in A$. Show that $\{f_n\}$ converges uniformly to f on A.

4. (Exercise 6.1.9) Let $f_n\colon [0,1] \to \mathbb{R}$ be a sequence of increasing functions (that is, $f_n(x) \geq f_n(y)$ whenever $x \geq y$). Suppose $f_n(0) = 0$ and $\lim_{n\to\infty} f_n(1) = 0$. Show that $\{f_n\}$ converges uniformly to 0.

5. (Exercise 6.1.10) Let $\{f_n\}$ be a sequence of functions defined on $[0,1]$. Suppose there exists a sequence of distinct numbers $x_n \in [0,1]$ such that

$$f_n(x_n) = 1.$$

Prove or disprove the following statements:

a) True or false: There exists $\{f_n\}$ as above that converges to 0 pointwise.

b) True or false: There exists $\{f_n\}$ as above that converges to 0 uniformly on $[0,1]$.