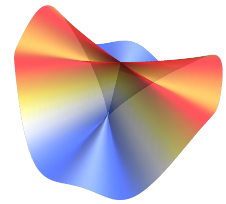
UC RIVERSIDE
Department of Mathematics

MATH 151B - Advanced Calculus



Homework 3

Due date: Monday, February 1 at 11:59pm

Instructions:

* You are encourage to work in group but you have to write your individual solution
* You can use any book, article or web-based mathematical material or computational software
* You can ask Chulan, Ryan, or Estela questions
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The homework needs to be typeset in “LaTeX” .

1. Find a closed form of the series

$$\sum_{k=2}^{\infty} kx^{k-2}$$

and the largest set on which this formula is valid.

2. (Exercise 6.2.21): Let $f_n(x) := \frac{x}{1+(nx)^2}$. Notice that f_n are differentiable functions.

   a) Show that $\{f_n\}$ converges uniformly to 0.

   b) Show that $|f_n'(x)| \leq 1$ for all x and all n.

   c) Show that $\{f_n'\}$ converges pointwise to a function discontinuous at the origin.

3. (Exercise 5.4.2): Let $b > 0, b \neq 1$ be given.

   a) Show that for every $y > 0$, there exists a unique number x such that $y = b^x$. Define the logarithm base b, $\log_b : (0,\infty) \to \mathbb{R}$, by $\log_b(y) := x$

   b) Show that $\log_b(x) = \frac{\ln(x)}{\ln(b)}$.

   c) Prove that if $c > 0, c \neq 1$, then $\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$.

   d) Prove $\log_b(xy) = \log_b(x) + \log_b(y)$, and $\log_b(x^y) = y\log_b(x)$

4. (Exercise 5.4.9): Using the logarithm find

$$\lim_{n\to\infty} n^{1/n}$$

Note: If you want to use L'Hopital's rule, you need to prove the result first. Alternatively, you can use directly the mean value theorem.

5. (Exercise 5.4.4): Use the geometric sum formula to show (for $t \neq -1$)

$$1 - t + t^2 - \cdots + (-1)^n t^n = \frac{1}{1+t} - \frac{(-1)^{n+1} t^{n+1}}{1+t}$$

Using this fact show

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

for all $x \in (-1, 1]$ (note that $x = 1$ is included). Finally, find the limit of the alternating harmonic series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - 1/2 + 1/3 - 1/4 + \cdots$$

6. (Exercise 5.4.8): Show that e^x is convex, in other words, show that if $a \leq x \leq b$, then

$$e^x \leq e^a \frac{b-x}{b-a} + e^b \frac{x-a}{b-a}$$