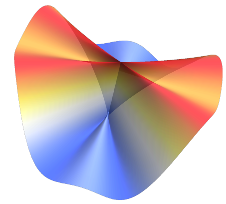
UC RIVERSIDE
Department of Mathematics

MATH 151B - Advanced Calculus



Homework 4

Due date: Tuesday, February 23 at 11:59pm

Instructions:

* You are encourage to work in group but you have to write your individual solution
* You can use any book, article or web-based mathematical material or computational software
* You can ask Chulan, Ryan, or Estela questions
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The homework needs to be typeset in "LaTeX" .

1. (Exercise 7.4.14): Prove the general Bolzano-Weierstrass theorem: Any bounded sequence $\{x_k\}$ in $\mathbb{R}^n$ has a convergent subsequence.

2. (Exercise 7.5.6): Prove the following version of the intermediate value theorem. Let (X,d) be a connected metric space and $f : X \to \mathbb{R}$ a continuous function. Suppose that there exist $x_0, x_1 \in X$ and $y \in \mathbb{R}$ such that $f(x_0) < y < f(x_1)$. Then prove that there exists $az \in X$ such that $f(z) = y$.

   Hint: See Exercise 7.5.5.

3. (Exercise 7.5.9): Take the metric space of continuous functions $C([0,1],\mathbb{R})$. Let $k : [0,1] \times [0,1] \to \mathbb{R}$ be a continuous function. Given $f \in C([0,1],\mathbb{R})$ define

$$\varphi_f(x) := \int_0^1 k(x,y) f(y) dy$$

   a) Show that $T(f) := \varphi_f$ defines a function $T : C([0,1],\mathbb{R}) \to C([0,1],\mathbb{R})$.
   b) Show that T is continuous.

4. (Exercise 8.3.1): Suppose $\gamma : (-1,1) \to \mathbb{R}^n$ and $\alpha : (-1,1) \to \mathbb{R}^n$ be two differentiable curves such that $\gamma(0) = \alpha(0)$ and $\gamma'(0) = \alpha'(0)$. Suppose $F : \mathbb{R}^n \to \mathbb{R}$ is a differentiable function. Show that

$$\frac{d}{dt}\bigg|_{t=0} F(\gamma(t)) = \frac{d}{dt}\bigg|_{t=0} F(\alpha(t))$$

5. (Exercise 8.3.10): Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable and suppose that whenever $x^2 + y^2 = 1$, then $f(x,y) = 0$. Prove that there exists at least one point (x_0, y_0) such that $\frac{\partial f}{\partial x}(x_0, y_0) = \frac{\partial f}{\partial y}(x_0, y_0) = 0$