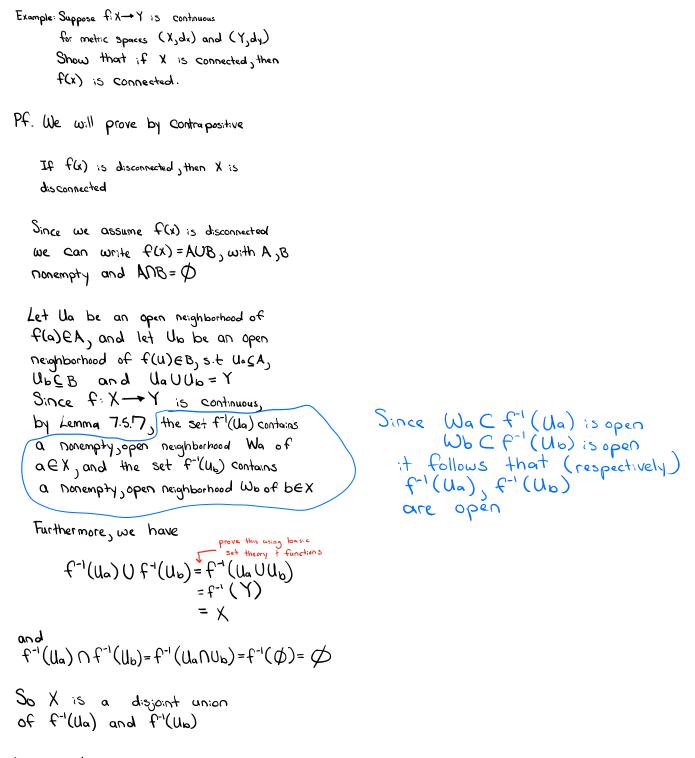
Definition:

```
Let (X, d) be a metric space.
A sequence $Xn3 is a <u>Cauchy</u>
Sequence if, for every \epsilon>0, there
exists MEN s.t. if n ≥ M and b ≥ M,
we have d(x_n, X_u) < \epsilon
            1X0-Xx1 < E"
Definition: Let (X,ol) be a metric
space we say that X is
Complete if every Cauchy sequence
Exn3 Converges to a point XEX
Example: Let C([a,b], IR) be the set of
real-valued functions on [a,b]. Define d
by d(f_{y}) = \sup_{x \in [a,b]} |f(x) - g(x)|
Show that C([a,b],IR) is a complete
metric space.
Proof: first Show d is a metric, consider f,g,h E C ([a,b], R)
   Non-negativity
                                                            Now we will show that C([a,b], R) is
   d(f_{3}g) = \sup |f(x) - g(x)|
        \geq |f(x) - g(x)|
                                                        🛪 Complete
         20
                                                           Let Efricant be a Cauchy sequence in C(Easb], IR)
d(f,g)=0 <> Sup | f(x)-g(x) =0 7
       <=> |f(x)-g(x)=0
                                                          We want to show that
                                                         Efrither Converges to its limit, say f.
Since Efrither is a Cauchy sequence of Functions,
       <=> f(x)-g(x)=0
                            for all XE [a,b]
           f(x) = q(x)
                                                          Efn(x) Sn=1 is a Cauchy sequence of numbers
                                                          for all XE [a,b]
 Symmetry_
 d(f,g) = Sup \{f(x) - g(x)\} : X \in D
                                                         By Prop 2.4.4 2fn(x) 3n=1 is also bounded
      =Sup|g(x)-f(x):xED
                                                         By the Bolzano-Werestrass Thereon (2.38)
      = d(g,f )
                                                        There exists a convergent sequence \hat{z}fn_i(y)_{i=1}
                                                         Converges to its limit f(x)
triangle inequality
                                                        Therefore,
d(f,h)=Sup |f-h]
      = Suplf-q+q-hl
                                                        |f_n(x) - f(x)| = |f_n(x) - f_{n_1}(x) + f_{n_2}(x) - f_{n_4}(x) + f_{n_4}(x) - f(x)|
      = Sup (1f-q1 + 1g-h1)
                                                                       \leq |f_n(x) - f_{n_i}(x)| + |f_{n_i}(x) - f_{n_k}(x)| + |f_{n_k}(x) - f(x)|
     ≤ Sup (lf-g1) + Sup(lg-h1)
      =d(f_{,q})+d(q_{,h})
                                                                                                 (=>2fn (X) is Cauchy
                                                                                                                   Since Efricard is
                                                                        < E/3 + E/3 + E/3
 =>d is a metric, and so
                                                                                                                   Convergent
                                                                        = 2
  C([a,b], R) is a metric
  Space
                                                      So Efn(x) 3 ... converges to f(x) for all x E Eagb]
                                                      Therefore, Efn3n=1 Converges to f in C([.b], R)
```



Namely, X is disconnected.