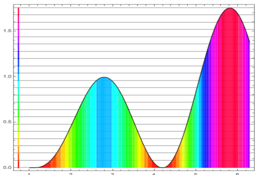
UC RIVERSIDE
Department of Mathematics

MATH 151C - Advanced Calculus



HIGHER STAKES HOMEWORK 1

Due dates:

Friday, May 14 at 11:59pm - One Early Bird Extra Credit Point
Sunday, May 16 at 11:59pm - Last Chance

Instructions:

* Work individually in the problems. You can ask questions to Estela, Chulan, or Ryan
* You can use any book, article or web-based mathematical material or computational software
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The homework needs to be typeset in "LaTeX" and uploaded through Gradescope in the iLearn Lecture page
* If a problem is similar to a problem in Hw #3 or Hw #4, you need to adapt the proof for this problem, not just refer or reproduce all the solution of the problem in the homework.

- **Problem 1:** Suppose $f : [0,1] \to \mathbb{R}$ is a measurable function. Show that the inverse image of a closed interval is a measurable set.

  Recall: the inverse image of a set C is

$$f^{-1}(C) = \{x \in [0,1] | f(x) \in C\}.$$

- **Problem 2:** Given a real number k, find an example of a sequence of Riemann integrable functions $f_n : [0,1] \to \mathbb{R}$ such that

  a) The sequence f_n converges pointwise to a non-Riemann integrable function $f : [0,1] \to \mathbb{R}$.

  b) f is a Lebesgue integrable function and $\displaystyle\int_{[0,1]} f = k$.

- **Problem 3:** Let f be a differentiable function on $[0,1]$. Prove that f' is measurable function.

  Hint: Consider

$$\lim_{n\to\infty} \frac{f\left(x+\frac{1}{n}\right)-f(x)}{\frac{1}{n}}.$$

- **Problem 4:** LLet $\{E_k\}$ be a sequence of Lebesgue measurable sets with

$$E_1 \supseteq E_2 \supseteq E_3 \supseteq \dots$$

Define the set E to be

$$E = \bigcap_{k=1}^{\infty} E_k \, .$$

If $m(E_1) < \infty$, show that

$$m(E) = \lim_{k \to \infty} m(E_k) \, .$$