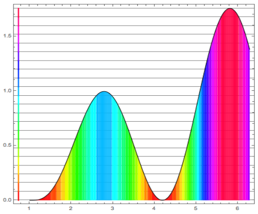
UC RIVERSIDE
Department of Mathematics

MATH 151C - Advanced Calculus



Homework 1

Due date: Tuesday, April 6 at 11:59pm

Instructions:

* You are encourage to work in group but you have to write your individual solution
* You can use any book, article or web-based mathematical material or computational software
* You can ask Ryan, or Estela questions
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The problems from the textbook will be indicated by the chapter and corresponding number.
* The homework needs to be typeset in “LaTeX” .

1. (Chapter 0, Problem 3) Let $f, g \in R[a,b]$ with $f(x) \leq g(x)$ for all $x \in [a,b]$. Prove that

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx.$$

2. (Chapter 0, Problem 5) Assume $f \in R[a,b]$.

a) Let $c \in [a,b]$. Suppose g is defined on $[a,b]$ and $g(x) = f(x)$ for all $x \neq c$. Show $g \in R[a,b]$.

b) Suppose g differs from f at a finite number of points. Show $g \in R[a,b]$.

c) Does this extend to the case where g and f differ at a countable number of points? Prove or give a counterexample.

3. (Chapter 0, Problem 6) Let $\{f_n\}$ be a sequence of functions with $f_n \in R[a,b]$ for each n. Suppose the sequence $\{f_n\}$ converges uniformly to f on $[a,b]$. Show that $f \in R[a,b]$.

4. (Chapter 0, Problem 7) Prove or modify and then prove: Let $f \in B[a,b]$. Define

$$f^{+}(x) = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f^{-}(x) = \begin{cases} 0 & \text{if } f(x) \geq 0 \\ -f(x) & \text{otherwise} \end{cases}$$

Then $f \in R[a,b]$ if and only if both $f^{+} \in R[a,b]$ and $f^{-} \in R[a,b]$.

5. (Chapter 0, Problem 11) Let $\{r_1, r_2, \ldots, r_n, \ldots\}$ be a counting of the rational numbers in the interval $[0,1]$. For each natural number k, define the function f_k by

$$f_k(x) = \begin{cases} 1 & \text{if } x \in \{r_1, r_2, \ldots, r_k\} \\ 0 & \text{otherwise} \end{cases}$$

a) Find f, the pointwise limit of the sequence $\{f_k\}$.

b) Show that $f_k \in R[0,1]$ for each k.

c) In general, if $\{f_n\}$ is a sequence of Riemann integrable functions which converge pointwise to f, is f Riemann integrable?